

Optimal Advertising Strategy for a Stackelberg Framework Under an Advertising-Driven Demand

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Abstract Advertising-driven demand is very common in practice. This paper considers pricing and advertising strategies issues in a two-echelon supply chain involving a manufacturer and a retailer. According to who undertakes the advertising expenditure, both the retailer-advertising case and the manufacturer-advertising case are analyzed under the Stackelberg framework. The crucial factor that affects the advertising strategy and the optimal profit for each participant is revealed. Furthermore, we compare profits of the two participants under different situations. It is demonstrated that both the manufacturer and the retailer gain more profits in the retailer-advertising case than those in the manufacturer-advertising case. In other words, the retailer has more incentive to advertise the product for the sake of maximizing its profit. Finally, a numerical illustration is presented to examine the change of the profit for each participant under different marginal demands and promotion degrees.

Keywords advertising-driven demand; Stackelberg game; retailer-advertising case; manufacturer-advertising case; pricing and advertising

1 Introduction

With the development of digital media, the mobile phone and the internet become the two most important approaches for retailers to promote their sales (Hu et al.^[1]). By using advertising, the expected market demand could be extended, which is crucial for all the participants in supply chains. Companies keep spending the bulk of their advertising budgets on building awareness of their products among innovators (Yenipazarli^[2]). Thus, the effect of advertising for extending consumers' demand is marked.

In competitive business environment, any retailer/business organization often searches the best strategy to capture the market (Sana^[3]). In order to enhance their revenue, both the manufacturer and the retailer have incentive to advertise their products. When the manufacturer undertakes the advertising cost, consumers' demand would be enhanced and the retailer will order more products from the manufacturer. When the retailer undertakes the advertising task, the expected market demand would be extended and the retailer will be endowed

by more power to make optimal decisions. In practice, both manufacturers and retailers may be the advertising undertaker. As we can see, handset makers like Apple and Huawei often advertise their products, which are manufacturer-advertising cases. In addition, e-commerce retailers like Tmall and JD also advertise some commodities to promote their sales, which are retailer-advertising cases.

Recent years, there are a lot of literatures related to advertising strategies in supply chains. Xie and Wei^[4] addressed channel coordination by seeking optimal cooperative advertising strategies and equilibrium on the premise that consumers' demand is price-and- advertising sensitive. Sethi, et al.^[5] proposed an optimal control model to obtain the optimal price and advertising effort over time, which is a dynamic optimization problem. Similarly, Krishnamoorthy, et al.^[6] considered dynamic advertising and pricing policies in a durable-good duopoly, and proposed an infinite-horizon model to capture dynamic price and advertising strategies. Zheng, et al.^[7] studied the optimal advertising and pricing decisions for luxury fashion brands in a market in the presence of different types of consumers. Chen^[8] considered pricing issues and cooperative advertising mechanisms on different sales channels in a supply chain, and showed quantitative insights into the interplay between upstream and downstream entities.

Many researches focus on two-echelon supply chains involving a manufacturer and a retailer. Yu, et al.^[9] considered the interact between the manufacturer and the retailer when pricing and advertising issues occurred simultaneously. Wu^[10] examined the retailer's platform goodwill and the manufacturer's brand goodwill under different channel structures in a two-layer supply chain. Noh, et al.^[11] constructed a Stackelberg game model for a two-layer supply chain and examined three different strategies under different power constructs. Farshbaf-Geranmayeh and Zaccour^[12] proposed a multi-period model to investigate pricing and advertising decisions in a supply chain, in the presence of two types of consumers. Sana^[13] constructed a structural model for a two-echelon supply chain consisting of a manufacturer and a retailer. Actually, the framework of this paper is similar to the above research, namely, a two-echelon supply chain involving only a manufacturer and a retailer. In addition, research on the design of the profit contract is also common when the manufacturer and the retailer choose to cooperate for advertising issues (see Sadigh, et al.^[14] and Xie, et al.^[15]).

In spite of the plenty literature with regard to pricing and advertising issues in two-layer supply chains, there are some research gaps in this area. Firstly, the crucial factor for determining pricing and advertising strategies in the advertising models has not been revealed. We show in this study that a relational expression consisting of the potential demand and the production cost per unit product is the crucial factor. Secondly, the problem that who has more incentive to advertise, the manufacturer or the retailer, has not been examined. We intend to acquire the optimal advertising strategy both for the manufacturer and the retailer. Clearly, the above two issues are of practical meanings for decision-makers. This paper intends to deal with these problems. Similar to Roy and Sana^[16], we consider the profit of the whole supply chain when choosing the optimal advertising strategy.

The rest of this paper is summarized as follows. Section 2 describes the necessary parameters and shows some reasonable assumptions. In Section 3, the retailer- advertising model is proposed, and the specialty of the solutions is analyzed. The manufacturer-advertising case is

discussed in Section 4. We demonstrate that the retailer always has more incentive to spend advertising expenditure than the manufacturer. Section 5 provides a numerical illustration to examine the change of the profit for each participant under different marginal demands and promotion degrees. Section 6 summarizes the paper.

2 Notations and Assumptions

In this paper, we discuss pricing and advertising decisions for a manufacturer and a retailer. The sales quantity is assumed to be sensitive to the sales price and the advertising expenditure. According to who undertakes the advertising cost, the retailer or the manufacturer, we divide the topic into two subjects. Only a single type of product is considered in the following discussion.

The notations used throughout the paper are given as follows:

Table 1 Notation definition

Parameters	Definition
a	the potential demand of the market
c	the production cost per unit product
w	the wholesale price determined by the manufacturer
p	the sales price given by the retailer
δ	the marginal demand with respect to the sales price
τ	the promotion degree of advertising on the sales quantity
Q	the sales quantity of the product
ϕ	the marginal cost of the promotion degree
$E(\tau)$	the advertising expenditure under promotion degree τ
π_m	the total profit of the manufacturer
π_r	the total profit of the retailer

Some supplementary instructions are presented for the above setting.

Following Sana^[17], the sales quantity is assumed to be price sensitive. Moreover, according to Jena and Ghadge^[18] and Jena, et al.^[19], the sales quantity is a linear function with respect to the promotion degree of advertising. Thus, we set the sales quantity function as follows: $Q = a + \tau - \delta p$. Besides, it is common that the firm's advertising cost is quadratic with respect to its advertising effort (see Wu, et al.^[20] and Lu, et al.^[21]). We also adopt this handling, and assume that

$$E(\tau) = \phi\tau^2/2.$$

Actually, the effect of the advertising is similar to the investment proposed by Zhang, et al.^[22].

We next present some reasonable assumptions for the sake of further discussion:

This paper only considers pricing and advertising issues of this product, without paying attention to the impact incurred by other substitutable products.

Because the considered model is deterministic, this paper doesn't consider stock out and over production issues.

In order to guarantee that the optimal price and the promotion degree of advertising are not infinite, we assume that $\delta\phi > 1/2$. In the following discussion, it will be used to guarantee the reasonability of solutions.

Actually, by analyzing the relations between parameters, we know that $\delta\phi$ is a dimensionless fitting parameter. It always holds in most literature.

In addition, $a - \delta c > 0$ always holds in almost all pricing-related literature (Lou, et al.^[23]). In the following discussion, we will show that $(a\delta c)$ is a crucial factor both for pricing strategies and for advertising strategies.

3 The Retailer-Advertising Model

This section discusses pricing and advertising issues in a two-echelon supply chain when the retailer undertakes the advertising cost. This is a Stackelberg game framework, which can be solved by the reverse recursive.

The decision model of the manufacturer is constructed as follows:

$$\begin{aligned} \max \quad & \pi_m = (w - c)(a + \tau - \delta p) \\ \text{s.t.} \quad & w \geq c. \end{aligned} \quad (1)$$

The model of the retailer is

$$\begin{aligned} \max \quad & \pi_r = (p - w)(a + \tau - \delta p) - \frac{1}{2}\phi\tau^2 \\ \text{s.t.} \quad & p \geq w, \\ & a + \tau - \delta p \geq 0. \end{aligned} \quad (2)$$

First, we solve the Stackelberg game involving (1) and (2) without considering their constraints.

By differentiating π_r , we have

$$\begin{cases} \frac{\partial \pi_r}{\partial p} = -2\delta p + \tau + a + \delta w = 0, \\ \frac{\partial \pi_r}{\partial \tau} = -\phi\tau + p - w = 0. \end{cases}$$

The solution of the above equation is given as follows:

$$\begin{cases} p = \frac{\phi a + (\delta\phi - 1)w}{2\delta\phi - 1}, \\ \tau = \frac{a - \delta w}{2\delta\phi - 1}. \end{cases} \quad (3)$$

The Hessian matrix of π_r is

$$H = \begin{bmatrix} -2\delta & 1 \\ 1 & -\phi \end{bmatrix}.$$

It is negative definite based on $\delta\phi > 1/2$. Hence, the solution given by (3) is the unique solution of model (2).

Substituting (3) into the objective function of model (1), we obtain

$$\pi_m = \frac{1}{2\delta\phi - 1} [-\delta^2\phi w^2 + (\delta\phi a + \delta^2\phi c)w - \delta\phi ac].$$

The solution of $\max \pi_m$ is obtained by setting the first-order derivative of π_m to be zero (marked by “*”):

$$w^* = \frac{a}{2\delta} + \frac{c}{2}. \quad (4)$$

By substituting (4) into (3), we have

$$p^* = \frac{3\delta\phi a - a + (\delta^2\phi - \delta)c}{2\delta(2\delta\phi - 1)} \quad (5)$$

and

$$\tau^* = \frac{a - \delta c}{2(2\delta\phi - 1)}. \quad (6)$$

Next, we examine the reasonability of the above obtained solutions. By $a - \delta c > 0$, it is obtained that

$$w^* - c = \frac{1}{2\delta}(a - \delta c) > 0.$$

Hence, the solution determined by (4) meets the constraint of model (1).

Similarly,

$$p^* - w^* = \frac{\delta\phi a - \delta^2\phi c}{2\delta(2\delta\phi - 1)} = \frac{\phi(a - \delta c)}{2(2\delta\phi - 1)} > 0$$

and

$$a + \tau^* - \delta p^* = \frac{\delta\phi(a - \delta c)}{2(2\delta\phi - 1)} > 0.$$

Thus, solutions given by (5) and (6) meet the constraints of model (2).

Besides, $\tau^* > 0$ also holds under $a - \delta c > 0$.

For the above derivation, we draw the following conclusion:

Proposition 1 *Under a certain pair (δ, ϕ) , the value of $(a - \delta c)$ is the unique factor for determining pricing and advertising strategies in the retailer-advertising model.*

By the acquired solutions, the optimal profit of the manufacturer is

$$\pi_m^* = \frac{\phi(a - \delta c)^2}{4(2\delta\phi - 1)} \quad (7)$$

and the optimal profit of the retailer is

$$\pi_r^* = \frac{\delta\phi^2(a - \delta c)^2}{4(2\delta\phi - 1)^2}. \quad (8)$$

4 The Manufacturer-Advertising Model

This section considers the situation in which the manufacturer undertakes the advertising expenditure. In practice, this type of case is also common. The aim is to compare the solutions and the profits of the two participants for the two situations. This is also a Stackelberg game framework.

The decision model of the manufacturer is constructed as follows:

$$\begin{aligned} \max \pi_m &= (w - c)(a + \tau - \delta p) - \frac{1}{2}\phi\tau^2 \\ \text{s.t. } w &\geq c, \\ a + \tau - \delta p &\geq 0. \end{aligned} \quad (9)$$

The model of the retailer is

$$\begin{aligned} \max \pi_r &= (p - w)(a + \tau - \delta p) \\ \text{s.t. } p &\geq w. \end{aligned} \quad (10)$$

Similar to the handling in the previous section, we first analyze the objective functions.

By differentiating π_r , we have

$$\frac{\partial \pi_r}{\partial p} = -2\delta p + a + \tau + \delta w.$$

The solution of $\max \pi_r$ is obtained as follows:

$$p = \frac{a + \tau + \delta w}{2\delta}. \quad (11)$$

By (11), the objective function of model (9) is transformed to

$$\pi_m = -\frac{\delta}{2}w^2 + \left(\frac{a}{2} + \frac{\tau}{2} + \frac{\delta c}{2}\right)w - \frac{1}{2}\phi\tau^2 - \frac{\tau c}{2} - \frac{ac}{2}. \quad (12)$$

By setting the first-order partial derivatives of π_m to be zero, we have

$$\begin{cases} \frac{\partial \pi_m}{\partial w} = -\delta w + \frac{a}{2} + \frac{\tau}{2} + \frac{\delta c}{2} = 0, \\ \frac{\partial \pi_m}{\partial \tau} = \frac{w}{2} - \frac{c}{2} - \phi\tau = 0. \end{cases}$$

The solution of the above equation is given as follows (marked by “’”):

$$\begin{cases} w' = \frac{2\phi a + (2\delta\phi - 1)c}{4\delta\phi - 1}, \\ \tau' = \frac{a - \delta c}{4\delta\phi - 1}. \end{cases} \quad (13)$$

By substituting (13) into (11), we obtain

$$p' = \frac{a}{2\delta} + \frac{a - \delta c}{2\delta(4\delta\phi - 1)} + \frac{2\phi a + (2\delta\phi - 1)c}{2(4\delta\phi - 1)}. \quad (14)$$

We examine the reasonability of the obtained solutions. By $a - \delta c > 0$, it is obtained that

$$p' - w' = \frac{\phi a - \delta\phi c}{4\delta\phi - 1} > 0.$$

Similarly,

$$w' - c = \frac{2\phi a - 2\delta\phi c}{4\delta\phi - 1} > 0$$

and

$$a + \tau' - \delta p' = \frac{\delta\phi(a - \delta c)}{4\delta\phi - 1} > 0.$$

Hence, the obtained solutions meet the corresponding constraints.

According to the above results, we draw the following conclusion:

Proposition 2 *Under a certain pair (δ, ϕ) , the value of $(a - \delta c)$ is also the unique factor for determining pricing and advertising strategies in the manufacturer-advertising model.*

By all the obtained solutions, the optimal profit of the manufacturer is

$$\pi'_m = \frac{(4\delta\phi^2 - \phi)(a - \delta c)^2}{2(4\delta\phi - 1)^2}. \quad (15)$$

And the optimal profit of the retailer is

$$\pi'_r = \frac{\delta\phi^2(a - \delta c)^2}{(4\delta\phi - 1)^2}. \quad (16)$$

5 Decision Analysis

By comparing advertising strategies of the two situations, it is obtained that

$$\tau' - \tau^* = \frac{a - \delta c}{4\delta\phi - 1} - \frac{a - \delta c}{2(2\delta\phi - 1)} < 0,$$

which means that the advertising expenditure in the manufacturer-advertising case is lower than the one in the retailer-advertising case.

Under $\delta\phi > 1/2$, we examine the difference of the profits for the two cases.

By comparing π_m^* and π'_m , we have

$$\pi_m^* - \pi'_m = \frac{\phi(a - \delta c)^2}{4(2\delta\phi - 1)} - \frac{\phi(a - \delta c)^2}{2(4\delta\phi - 1)} = \phi(a - \delta c)^2 \left[\frac{1}{8\delta\phi - 4} - \frac{1}{8\delta\phi - 2} \right] > 0.$$

By comparing π^* and π' , we have

$$\pi_r^* - \pi'_r = \frac{\delta\phi^2(a - \delta c)^2}{4(2\delta\phi - 1)^2} - \frac{\delta\phi^2(a - \delta c)^2}{(4\delta\phi - 1)^2} = \frac{\delta\phi^2(a - \delta c)^2}{16\delta^2\phi^2 - 16\delta\phi + 4} - \frac{\delta\phi^2(a - \delta c)^2}{16\delta^2\phi^2 - 8\delta\phi + 1} > 0.$$

According to the above results, we present the following conclusion:

Proposition 3 *Both the manufacturer and the retailer gain more profits in the retailer-advertising case than in the manufacturer-advertising case.*

According to Proposition 3, the retailer always has more incentive to undertake advertising expenditure than the manufacturer.

We examine the impact of the crucial factor $(a - \delta c)$ over profits of the manufacturer and the retailer.

According to (15), we have

$$\frac{d\pi'_m}{d(a - \delta c)} = \frac{(4\delta\phi^2 - \phi)(a - \delta c)}{(4\delta\phi - 1)^2}.$$

According to (16), we have

$$\frac{d\pi'_r}{d(a - \delta c)} = \frac{2\delta\phi^2(a - \delta c)}{(4\delta\phi - 1)^2}.$$

Given that $\delta\phi > 1/2$, we show the following result:

$$\frac{d\pi'_m}{d(a-\delta c)} - \frac{d\pi'_r}{d(a-\delta c)} = \frac{(a-\delta c)(2\delta\phi^2 - \phi)}{(4\delta\phi - 1)^2} > 0.$$

Hence, the profit of the manufacturer is more sensitive with respect to $(a - \delta c)$ than the profit of the retailer.

Further, we examine the possibility of the cooperative case for the retailer-advertising model, in which the manufacturer shares a portion of the advertising expenditure so as to enhance their profits.

We denote by k the award for unit promotion degree of advertising. Thus, the decision model of the manufacturer is constructed as follows:

$$\begin{aligned} \max \pi_m &= (w - c)(a + \tau - \delta p) - k\tau \\ \text{s.t. } w &\geq c. \end{aligned} \quad (17)$$

And the decision model of the manufacturer is

$$\begin{aligned} \max \pi_r &= (p - w)(a + \tau - \delta p) - \frac{1}{2}\phi\tau^2 + k\tau \\ \text{s.t. } p &\geq w, \\ a + \tau - \delta p &\geq 0. \end{aligned} \quad (18)$$

By differentiating π_r , we have

$$\begin{cases} \frac{\partial \pi_r}{\partial p} = -2\delta p + \tau + a + \delta w = 0, \\ \frac{\partial \pi_r}{\partial \tau} = -\phi\tau + p - w + k = 0. \end{cases}$$

The solution of the above equation is given as follows:

$$\begin{cases} p = \frac{\phi a + (\delta\phi - 1)w + k}{2\delta\phi - 1}, \\ \tau = \frac{a - \delta w + 2\delta k}{2\delta\phi - 1}. \end{cases} \quad (19)$$

The Hessian matrix of π_r is negative definite based on $\delta\phi > 1/2$. Hence, the solution given by (19) is the unique solution of model (18).

Substituting (19) into the objective function of model (17), we obtain

$$\pi_m = \frac{-\delta^2\phi w^2 - 2\delta k^2 + 2\delta wk + (\delta\phi a + \delta^2\phi c)w - (a + \delta c)k - \delta\phi ac}{2\delta\phi - 1}.$$

By differentiating π_m , we have

$$\begin{cases} \frac{\partial \pi_m}{\partial w} = -2\delta^2\phi w + 2\delta k + \delta\phi a + \delta^2\phi c = 0, \\ \frac{\partial \pi_m}{\partial k} = -4\delta k + 2\delta w - a - \delta c = 0. \end{cases}$$

By solving the above equation, we have:

$$\begin{cases} w = \frac{a}{2\delta} + \frac{c}{2}, \\ k = 0. \end{cases} \quad (20)$$

The Hessian matrix of π_m is also negative definite based on $\delta\phi > 1/2$. Hence, the solution given by (20) is the unique solution of $\max \pi_m$.

Given the above result, we draw the following conclusion:

Proposition 4 *The manufacturer has no motivation to share the advertising expenditure.*

6 Numerical Illustrations

This section provides a numerical illustration to examine the change of the profit for each participant under different marginal demands and promotion degrees so as to analyze the parameter sensitivity.

Consider the following scenario: $a = 2000$, $c = 500$, $\delta \in [1, 2]$, $\phi \in [1, 2]$. Clearly, the given parameters meet $a - \delta c > 0$ and $\delta\phi > 1/2$.

First, we consider the profit of the manufacturer under the two different situations. By the given parameters, the two functions are acquired according to (7) and (15) as follows:

$$\pi_m^* = \frac{(2000 - 500\delta)^2\phi}{8\delta\phi - 4}$$

and

$$\pi'_m = \frac{(2000 - 500\delta)^2\phi}{8\delta\phi - 2}.$$

We draw the following curved surface graph by MATLAB as follows:

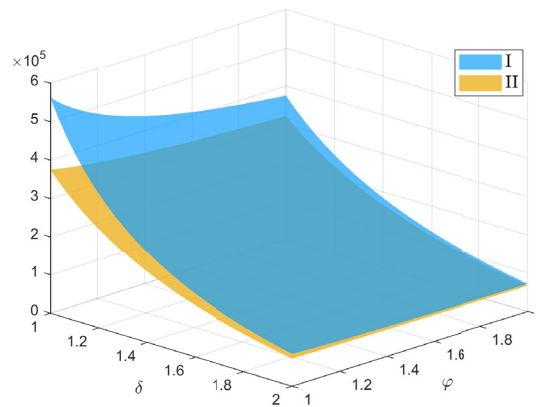


Figure 1 Curved surfaces of the manufacturer's profits

In Figure 1, I represents the profit function in the retailer-advertising case, and II represents the profit function in the manufacturer-advertising case.

It is easy to find that the difference between the profits is small when δ and ϕ are both large.

Next, we turn our attention to the profit of the manufacturer under the two different situations. By the given parameters, the two functions are acquired according to (8) and (16) as follows:

$$\pi_r^* = \frac{\delta\phi^2(2000 - 500\delta)^2}{4(2\delta\phi - 1)^2}$$

and

$$\pi_r' = \frac{\delta\phi^2(2000 - 500\delta)^2}{(4\delta\phi - 1)^2}.$$

We draw the following curved surface graph as follows:

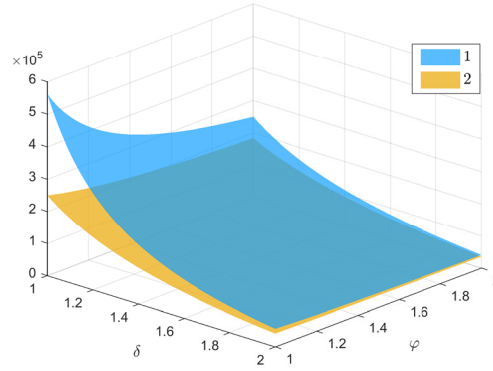


Figure 2 Curved surfaces of the retailer's profits

In Figure 2, 1 represents the profit function in the retailer-advertising case, and 2 represents the profit function in the manufacturer-advertising case.

Actually, it is shown by Figures 1 and 2 that both the manufacturer and the retailer gain more profits when the retailer advertises the product, which coincides with Proposition 3.

Similar to the change of the manufacturer's profit functions, the difference between the profits is small when δ and ϕ are both large. In addition, by comparing Figures 1 and 2, it is shown that the profit of the retailer in the retailer-advertising case is more sensitive to δ and ϕ than other profit functions.

Finally, we analyze the sensitivity of the promotion degree of advertising. According to (6), the optimal decision of the promotion degree is

$$\tau = \frac{2000 - 500\delta}{4\delta\phi - 2}.$$

We draw the following curved surface graph as follows:

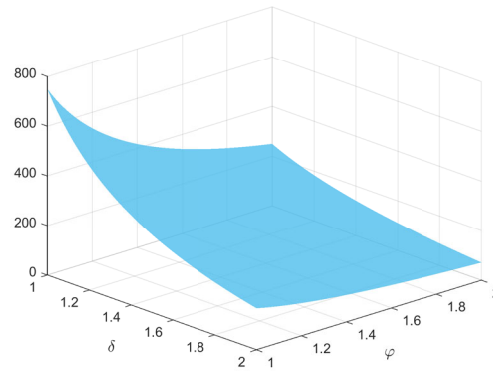


Figure 3 Curved surface of the promotion degree of advertising

According to Figure 3, the promotion degree has negative correlations with both the marginal demand and the marginal cost of the promotion. When the value of δ (or ϕ) is low, the sensitivity of the promotion degree of advertising is strong.

7 Conclusions

In this paper, we investigate pricing and advertising decision issues in a two-layer supply chain under a price-and-advertising sensitive demand. Both the manufacturer-advertising case and the retailer-advertising case are discussed. The change of the profit for each participant under different marginal demands and promotion degrees is examined.

The managerial insights of this research are summarized as follows: 1) By showing the crucial impact of $(a - \delta c)$ over the advertising strategy, we are able to judge the effect of advertising before the selling period begins. 2) It is shown that the retailer is the optimal candidate for undertaking the advertising expenditure. Hence, the retailer is endowed with more bargaining power when the manufacturer seeks for cooperation. Given that the retailer promotes the demand, she could ask for some compensations.

There are some shortcomings in our research. In our model, only a manufacturer and a retailer are involved. In the further study, substituting products and competitive participants will be considered. Moreover, given that the retailer plays a dominant role in advertising, cooperation and contract should be taken into consideration to improve the present model in the following investigation.

Data Availability Statement

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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