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A Superior Sales Mode for a Two-Echelon Supply Chain of Perishable Products

Mengqi BAO
School of Management, Wenzhou Business College, Wenzhou 325035*, China*
E-mail: *baomengq@126.com*

Xiaozhen DAI*
School of Management, Wenzhou Business College, Wenzhou 325035*, China*
E-mail: *daixz81@126.com*

Abstract This paper considers sales mode selection issues for a two-echelon supply chain of perishable products involving a supplier and a retailer. Consumers' purchase desire is assumed to be positive correlation with the freshness of the product. First, the traditional sales mode is examined, in which the supplier and the retailer play a Stackelberg game. We propose a three-layer decision model for this situation, and obtain dynamic pricing strategies and selling cycle length. It is shown that the retailer has little motivation to order many perishable products so as to avoid a long selling cycle length. Second, the commission-charge mode is analyzed, in which the retailer declares its decision first. In this mode, we demonstrate that the perishable product will be on sale during the whole shelf life under a certain condition. The correlation between the sales price of each stage and the remaining shelf-life length is analyzed. Third, the superiority analysis for the two sales modes is conducted. We show the relation between the selling cycle lengths of the two modes. By our analysis, it is shown that both the supplier and the retailer gain more profits when the commission-charge mode is adopted and the commission rate locates in a certain open interval. Finally, a numerical illustration is presented to visualize the discussed models, and some supplements are made for the acquired conclusions by the illustration.

Keywords pricing; sales mode; commission-charge mode; selling cycle length; superiority analysis

1 Introduction

The pricing of perishable products was proposed by Weatherford and Bodily[1] to deal with items possessing a time-sensitive property. Because of the time-varying quality of perishable products, retailers, or service providers, always price their products dynamically (Chatwin[2]; Ahmadi, et al.[3]). For consumers, both quality and sales price affect their purchase behavior. Hence, the market demand is often formulated as a binary function of time and price.

As is known, the shelf-life length of a perishable product is crucial when a retailer makes dynamic pricing decisions. In practice, a perishable product may not be on sale all the time throughout its shelf life. Study on how to determine the optimal selling cycle length of a

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* Corresponding author

perishable item is of practical significance. Apparently, a perishable product with a very limited remaining shelf life will yield less profit. Hence, the retailer seldom orders many perishable products in order to avoid a long selling period. Given the above, one key issue for the seller is to determine the optimal selling cycle length for the perishable products.

The traditional sales mode for a two-echelon supply chain is that a supplier (or a manufacturer) wholesales products to a retailer. With the increasing popularity of online retailing, the commission-charge mode is widely adopted in recent years (Chen, et al.[4]). In practice, the commission mode is also used in physical stores, such as Wal-Mart. Under different sales modes, decision orders and profits of participants may be different, as well as the sales price. Sales mode choice is a matter of priority for a supply chain before a selling cycle starts.

This paper is concerned with sales mode choice of a two-layer perishable-product supply chain involving one supplier and one retailer. The retailer is endowed with the right to select the sales mode. In the traditional mode, the retailer determines the sales price of each stage and the selling cycle length. Comparatively, the supplier determines these decision variables in the commission-charge mode, and shares profits with the retailer. The superiority of the two sales modes will be examined to ascertain which is better for both the supplier and the retailer. In addition, we analyze how the market demand influences the selling cycle length, and verify some critical conditions.

The remainder of this paper is organized as follows. Section 2 briefly reviews the related literature about optimal pricing of perishable products. In Section 3, we show necessary notations and give some assumptions. Section 4 conducts a Stackelberg game consisting of a supplier and a retailer. Section 5 proposes a commission-pricing model. In Section 6, the superiority of the two sales mode is examined. Section 7 presents a numerical illustration to verify the obtained conclusions and make some supplements. Section 8 summarizes the whole paper.

2 Literature Review

As is well known, pricing decisions play an important role in supply chain management. There is a growing body of literature focusing on optimal pricing of perishable products. In this section, we analyze similarities and differences between this research and existing literature to show our motivation and innovation.

Quality plays a crucial role in pricing decisions of perishable products. Because of the variable quality of perishable products, dynamic pricing strategies are often adopted to enhance the profits of sellers. Schlosser and Boissier[5] pointed out that multiple times of adjustment on the sales price is necessary for the retailer. Herbon and Khmelnitsky[6] developed a dynamic pricing model for perishable products, and showed the optimal replenishment schedule and pricing strategies over time. Lu, et al.[7] proposed several dynamic pricing models by considering replenishment cycle length and preservation technology investment. Dye[8] presented a deterministic multi-period setting, and proposed a dynamic pricing model based on sales price and freshness index.

In our research, a certain type of perishable product with quality deterioration is considered. Some meaningful articles have been published with regard to this aspect. Chen, et al.[9] analyzed dynamic pricing issues of deteriorating products, and drew the conclusion that optimal prices of deteriorating products consistently decrease over time. Fan, at al.[10] proposed a dynamic pricing strategy for multi-batch fresh produce by solving a dynamic programming model of revenue management. Chen, et al.[11] investigated the influence of freshness-keeping effort on the optimal pricing strategy for a batch of a perishable product with a stochastic freshness-keeping ratio. Kaya and Bayer[12] proposed a decreasing demand function with respect to the age of perishable products, which is similar to our setting.

A retailer who sells perishable products often pays a lot of attention to the shelf-life length. Haijema[13] considered a perishable product with short shelf-life, which is quite common in practice. Sainathan[14] considered a perishable product with a two-period shelf life, and proposed optimal pricing strategies for different situations under dynamic demand substitution among customers. Moreover, Chen, et al.[15] and Muriana[16] both paid much attention to perishable products with fixed shelf life, which is also our research object in this paper. Since freshness can improve customer satisfaction, Liang, et al.[17] examined different values of shelf-life length for the perishable product. Keskin, et al.[18] considered joint pricing and inventory ordering decisions over a finite time horizon of T periods. Similar to Avinadav, et al.[19], we aim to determine the optimal selling cycle length for a perishable product with a fixed shelf-life length. Similar to Zarouri, et al.[20], we adopt the price markdown policy to deal with the sales of perishable products.

Motivated by the above research, we conduct this study. The framework of this paper is the same as Liu, et al.[21], involving a manufacturer and a retailer. By reviewing existing literature, we find that there are some crucial issues need to be addressed. To the best of our knowledge, little literature pays attention to the commission-charge mode of perishable products till now except for Song, et al.[22], who considered proportional and fixed commissions on the third-party channels for perishable products. In reality, retailers often adopt this sales mode. It is a research gap in theory. We will examine both the traditional sales mode and the commission mode to find out a better one for both the supplier and the retailer. As our main contribution, the impacts of the market demand and the shelf-life length over selling cycle length will be revealed in this study. We show the change rule of the selling cycle length. Moreover, the sensitivity of the profit with respect to the shelf-life length is presented.

3 Assumptions and Notations

This paper deals with sales-mode choice issues for a two-echelon perishable- product supply chain involving a supplier and a retailer. Dynamic pricing strategies and selling cycle lengths are discussed during the exploration. Similar to Levin, et al.[23], the sales cycle is treated to be discrete, and the perishable product is dynamic priced so as to maximize the seller's profit. In the traditional mode, a supplier and a retailer play a Stackelberg game to maximize their own profit. In the commission-charge mode, the supplier determines the sales price of each stage after the retailer declares a commission rate. According to the above description, we make a reasonable assumption that it is the retailer who determines the type of the sales mode for this two-layer supply chain.

Similar to many other deterministic models, supply constraint is not considered in this study. Both the two participants in this discussion are assumed to be able to acquire complete information during their decision processes. Moreover, we take no account of replenishment issues, which means that all the perishable products are produced on the same date.

The notations used in the following discussion are given in Table 1.

Table 1 Model parameters

Parameters	Definition
a	The potential market demand of each isometric stage
l	The length of each sales stage
c	The cost of the supplier per unit item
δ	The linear price-sensitive coefficient of the demand quantity
T	The shelf-life length of the considered product
t	The tth sales stage, $1 \leq t \leq T$, $t \in \mathrm{Z}^+$
p_t	The retail price at stage t
D_t	The demand (sales) quantity at stage t, $D_t = a - \delta p_t - (t-1)a/T$
w	The wholesale price in the traditional mode
β	The commission rate in the commission-charge mode, $\beta \in (0, 1)$
Ω	The set of all reasonable β
m	The number of sales stages for the traditional mode, $m \in \mathrm{Z}^+$
n	The number of sales stages for the commission mode, $n \in \mathrm{Z}^+$
$G\langle\rangle$	Gauss rounding function for determining the selling cycle length under the discrete scenario
π_1	The total profit of the supply chain in the traditional mode
$\pi_{1,s}$	The profit of the supplier in the traditional mode
$\pi_{1,r}$	The profit of the retailer in the traditional mode
π_2	The total profit of the supply chain in the commission-charge mode
$\pi_{2,s}$	The profit of the supplier in the commission mode, $\pi_{2,s} = (1-\beta)\pi_2$
$\pi_{2,r}$	The profit of the retailer in the commission mode, $\pi_{2,r} = \beta\pi_2$

We make some explanations for the above setting.

Similar to Lou, et al.[24], the sales quantity function D_t, which is also called the demand function, is formulated as an additive combination of p_t and t. Actually, with the decrease of the freshness, consumers' purchase desire for this batch of products decreases accordingly. This formula guarantees that for any $p_t \geq 0$, D_t is no longer positive when t exceeds the shelf-life T, which coincides with reality.

In practice, many perishable products may be on the shelf for a short period compared with its shelf-life T. Given that $D_t = 0$ when $t > T$, we know that $ml \leq T$ and $nl \leq T$ hold. In the following discussion, the optimal values of m and n will be examined.

Following Lou, et al.[25], $G\langle\rangle$ is Gauss rounding function, by which the number of sales stages can be determined. This paper mainly considers situations in which $m \geq 2$ and $n \geq 2$.

Under the assumption that the length of each stage $l = 1$ (following Liu, et al.[26]), the number of sales stages is equal to the selling cycle length of the product, which will be used interchangeably.

4 The Traditional Sales Mode

This section conducts a Stackelberg model for the traditional sales mode to achieve the optimal sales price of each stage and the number of sales stages. The supplier determines the wholesale price w, and then the retailer determines the sales price p_t of each stage and the number of sales stages m.

Clearly, the supplier concerns the total ordering quantity of the retailer, rather than the sales quantity of the retailer at each stage. Based on this analysis, the decision model of the supplier is formulated as follows:

$$\begin{aligned} &\max \pi_{1,s} = (w-c)\sum_{t=1}^{m}\left[a-\delta p_t-\frac{(t-1)}{T}a\right] \\ &\text{s.t. } w \geq c. \end{aligned} \tag{1}$$

In model (1), the constraint of the ordering quantity is not considered, as

$$a-\delta p_t-\frac{(t-1)}{T}a \geq 0$$

for each stage will be guaranteed by the retailer.

The total profit of the retailer is the accumulation of the profit of each sales stage. Hence, the decision model of the retailer is

$$\begin{aligned} &\max \pi_{1,r} = \sum_{t=1}^{m}(p_t-w)\left[a-\delta p_t-\frac{(t-1)}{T}a\right] \\ &\text{s.t. } p_t \geq w \\ &\quad a-\delta p_t-\frac{(t-1)}{T}a \geq 0,\ \forall t \in \{1,\cdots,m\}. \end{aligned} \tag{2}$$

Clearly, model (1) and model (2) constitute a Stackelberg game. The objective function of model (2) shows that p_t and m are two independent variables. Hence, from the perspective of the supplier, analyzing the expression of p_t is the first-line task. As the follower in this game, the retailer will handle the sales price of each stage p_t and the number of sales stages $m$sequentially after acquiring the expressions of w.

According to the above discussion, the Stackelberg game is a three-stage decision problem. First, we deal with the two models without considering their constraints.

Noticing that the objective function of model (2) is an accumulation of profits of m stages, we denote by $\pi_{1,r,t}$ the profit of the retailer at stage t. According to the given parameters, $\pi_{1,r,t}$ is shown as follows:

$$\pi_{1,r,t} = -\delta p_t^2 + \left[a-\frac{(t-1)}{T}a+\delta w\right]p_t - aw + \frac{(t-1)}{T}aw.$$

The following equation is obtained by differentiating $\pi_{1,r,t}$ with regard to p_t:

$$\frac{d\pi_{1,r,t}}{dp_t} = -2\delta p_t + a - \frac{(t-1)}{T}a + \delta w = 0.$$

It is obtained that

$$p_t = \frac{a}{2\delta} + \frac{w}{2} - \frac{(t-1)}{2\delta T}a. \tag{3}$$

Because

$$\frac{d^2\pi_{1,r,t}}{dp_t^2} = -2\delta < 0,$$

the solution determined by (3) is the unique solution of max $\pi_{1,r,t}$.

By (3), the sales quantity at stage t is acquired as follows:

$$D_t = \frac{a}{2} - \frac{\delta w}{2} - \frac{(t-1)}{2T}a.$$

And the total sales quantity of the retailer is

$$\sum_{t=1}^{m} D_t = \frac{ma}{2} - \frac{\delta mw}{2} - \frac{m(m-1)a}{4T}. \tag{4}$$

By (4), the objective function of model (1) is obtained as follows:

$$\pi_{1,s} = -\frac{\delta m}{2}w^2 + \left[\frac{ma}{2} - \frac{m(m-1)a}{4T} + \frac{\delta cm}{2}\right]w - \frac{mac}{2} + \frac{m(m-1)ac}{4T}.$$

The following equation is obtained by differentiating $\pi_{1,s}$:

$$\frac{d\pi_{1,s}}{dw} = -\delta mw + \frac{ma}{2} - \frac{m(m-1)a}{4T} + \frac{\delta cm}{2} = 0.$$

We use "*"to represent optimal solutions. It is obtained that

$$w^* = \frac{a}{2\delta} - \frac{(m-1)a}{4\delta T} + \frac{c}{2}. \tag{5}$$

According to

$$\frac{d^2\pi_{1,s}}{dw^2} = -\delta m < 0,$$

the value given by (5) is the unique solution of max $\pi_{1,s}$.

By substituting (5) into (3), we have

$$p_t^* = \frac{3a}{4\delta} + \frac{c}{4} - \frac{(m-1)a}{8\delta T} - \frac{(t-1)a}{2\delta T}. \tag{6}$$

Actually, (6) can be transformed to

$$p_t^* = \frac{a}{4\delta} + \frac{c}{4} - \frac{(m-1)a}{8\delta T} + \frac{a}{2\delta T} + \frac{(T-t)a}{2\delta T},$$

which means that p_t^* and the remaining shelf life $(T-t)$ have a linear positive correlation.

By the same way, the sales quantity at stage t is obtained as follows:

$$D_t^* = \frac{a}{4} - \frac{\delta c}{4} + \frac{(m-1)a}{8T} - \frac{(t-1)a}{2T}. \tag{7}$$

And the total sales quantity of the retailer is

$$\sum_{t=1}^{m} D_t^* = \frac{ma}{4} - \frac{\delta mc}{4} - \frac{m(m-1)a}{8T}. \tag{8}$$

Next, we deal with the optimal m. Clearly, (7) implies that D_t is a strictly decreasing function with respect to t. Thus, we only need to guarantee $D_m \geq 0$. By Gauss rounding function, the optimal number of sales stages denoted by m_o is obtained as follows:

$$m_o = G\left\langle \max m : \frac{a}{4} - \frac{\delta c}{4} - \frac{3(m-1)a}{8T} \geq 0 \right\rangle. \tag{9}$$

According to (9), the selling cycle length is positively correlated to the shelf-life length. Moreover, the upper limit of the selling cycle length is $(a/4 - \delta c/4)$.

Apparently, (9) guarantees that m_o meets the corresponding constraint in model (2). By (9), it is obtained that

$$\frac{a}{4} - \frac{3(m_o-1)a}{8T} > 0,$$

i.e.,

$$m_o - 1 < \frac{2}{3}T.$$

Further, we show the change rule of m_o when a varies.

Following the above symbols, m_o is a critical point under some given a, i.e.,

$$\frac{a}{4} - \frac{\delta c}{4} - \frac{3(m_o-1)a}{8T} = 0, \tag{10}$$

which suggests that the optimal number of sales stages definitely decreases when a reduces.

Consider the following inequality set:

$$\begin{cases} \dfrac{a-\Delta a}{4} - \dfrac{\delta c}{4} - \dfrac{3(m_o-1)(a-\Delta a)}{8T} < 0, \\ \dfrac{a-\Delta a}{4} - \dfrac{\delta c}{4} - \dfrac{3(m_o-2)(a-\Delta a)}{8T} \geq 0. \end{cases} \tag{11}$$

By using (10), (11) is transformed to

$$\begin{cases} \dfrac{(3m_o-3-2T)\Delta a}{8T} < 0, \\ \dfrac{(3m_o-6-2T)\Delta a + 3a}{8T} \geq 0. \end{cases}$$

By figuring out the above inequality set, we have

$$\Delta a \in \left(0, \frac{3a}{2T-3m_o+6}\right].$$

Namely, the optimal number of sales stages is equal to $m_o - 1$ when the demand decreases to $a - \Delta a$. This result reveals the rate of change for the number of sales stages.

We then examine the obtained solutions to verify whether they meet the constraints of models.

The inequality

$$\frac{a}{4} - \frac{\delta c}{4} - \frac{3(m_o-1)a}{8T} \geq 0$$

is equivalent to

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{3(m_o-1)a}{4T} \geq 0.$$

For the wholesale price w^*, we have

$$w^* - c = \frac{a}{2\delta} - \frac{(m_o-1)a}{4\delta T} - \frac{c}{2} \geq \frac{1}{\delta}\left[\frac{a}{2} - \frac{\delta c}{2} - \frac{3(m_o-1)a}{4T}\right] \geq 0.$$

For the sales price at stage t ($t \leq m_o$), we have

$$p_t^* - w^* = \frac{a}{4\delta} - \frac{c}{4} + \frac{(m_o-1)a}{8\delta T} - \frac{(t-1)a}{2\delta T} \geq \frac{1}{\delta}\left[\frac{a}{4} - \frac{\delta c}{4} - \frac{3(m_o-1)a}{8T}\right] \geq 0.$$

Hence, the obtained solutions meet the constraints of model (1) and model (2).

For the optimal number of sales stages given by (9), we have the following proposition for the traditional sales mode:

Proposition 1 The selling cycle length is shorter than the shelf-life length when the shelf-life length is longer than 3 stages.

Proof Owing to the property of the strictly decreasing of $f(m)$:

$$f(m) = \frac{a}{4} - \frac{\delta c}{4} - \frac{3(m-1)a}{8T},$$

we just need to verify whether $f(T) < 0$ when $T \geq 3$.

Given the above, we have

$$f(T) = \frac{a}{4} - \frac{\delta c}{4} - \frac{3(T-1)a}{8T} = \frac{3a - Ta}{8T} - \frac{\delta c}{4} \leq -\frac{\delta c}{4} < 0,$$

which bears out the proposition. In other words, the optimal number of sales stages m_o must be shorter than the shelf-life length T when $T \geq 3$.

Proposition 1 actually tells the fact that the selling cycle length of a perishable product generally cannot be as long as its shelf-life length T in the traditional mode. Hence, the retailer has little motivation to order too many perishable products so as to avoid a long selling cycle length.

By (5) and (8), the profit of the supplier is obtained as follows:

$$\pi_{1,s}^* = \frac{a^2}{32\delta T^2}m_o^3 - \left(\frac{a^2}{8\delta T} + \frac{a^2}{16\delta T^2} - \frac{ac}{8T}\right)m_o^2 + \frac{(2aT - 2\delta cT + a)^2}{32\delta T^2}m_o. \tag{12}$$

By (5) $\sim$ (7) and

$$\sum_{t=1}^{m} t^2 = \frac{m_o(m_o+1)(2m_o+1)}{6},$$

the profit of the retailer is acquired as follows:

$$\begin{aligned}\pi_{1,r}^* =& \frac{7a^2}{192\delta T^2}m_o^3 - \left(\frac{a^2}{32\delta T^2} + \frac{a^2}{16\delta T} - \frac{ac}{16T}\right)m_o^2 \\ &+ \left(\frac{a^2}{16\delta T} - \frac{ac}{16T} - \frac{a^2}{192\delta T^2} + \frac{a^2}{16\delta} + \frac{\delta c^2}{16} - \frac{ac}{8}\right)m_o.\end{aligned} \tag{13}$$

Actually, m_o is a discrete function with respect to a. Hence, both (12) and (13) can be regarded as univariate functions of a under given δ, c and T.

5 The Commission-Charge Mode

This section considers the commission-charge sales mode, in which the supplier makes decisions for sales prices and the number of sales stages, and the retailer determines the commission rate. Different from the previous section, the retailer is the one who first makes decisions. In practice, many supermarkets like Wal-Mart adopt the commission mode to sell products.

By the given parameters, the decision model of the supplier is constructed as follows:

$$\begin{aligned} &\max \pi_{2,s} = (1-\beta)\sum_{t=1}^{n}(p_t - c)\left[a - \delta p_t - \frac{(t-1)}{T}a\right] \\ &\text{s.t. } p_t \geq c \\ &\qquad a - \delta p_t - \frac{(t-1)}{T}a \geq 0,\ \forall t \in \{1,\cdots,n\}. \end{aligned} \tag{14}$$

And the decision model of the retailer is

$$\begin{aligned} &\max \pi_{2,r} = \beta\sum_{t=1}^{n}(p_t - c)\left[a - \delta p_t - \frac{(t-1)}{T}a\right] \\ &\text{s.t. } p_t \geq c \\ &\qquad a - \delta p_t - \frac{(t-1)}{T}a \geq 0,\ \forall t \in \{1,\cdots,n\}. \end{aligned} \tag{15}$$

Clearly, the commission rate β does not affect the supplier to make decisions. Thus, the first step is to analyze decisions of the supplier. Similar to the previous section, p_t and n are two independent variables, which means that we can solve this problem successively.

We denote by

$$\pi_{2,t} = (p_t - c)\left[a - \delta p_t - \frac{(t-1)}{T}a\right]$$

the profit of the whole supply chain at stage t. By differentiating $\pi_{2,t}$, we have

$$\frac{d\pi_{2,t}}{dp_t} = -2\delta p_t + a - \frac{(t-1)}{T}a + \delta c = 0.$$

We use "$'$" to represent optimal solutions in this section. It is easy to obtain

$$p_t' = \frac{a}{2\delta} + \frac{c}{2} - \frac{(t-1)a}{2\delta T}. \tag{16}$$

According to

$$\frac{d^2\pi_{2,t}}{dp_t^2} = -2\delta < 0,$$

the value given by (16) is the unique solution of max $\pi_{2,t}$.

Actually, (16) can be transformed to

$$p_t' = \frac{a}{2\delta T} + \frac{c}{2} + \frac{(T-t)a}{2\delta T},$$

which means that p_t' and the remaining shelf life $(T-t)$ have a linear positive correlation. Clearly, this conclusion is the same as the one in the traditional sales mode.

By (16), the sales quantity at stage $t$is acquired as follows:

$$D_t' = \frac{a}{2} - \frac{\delta c}{2} - \frac{(t-1)a}{2T}. \tag{17}$$

As it is shown by (17), D'_t has nothing to do with the number of sale stages, which is different from the situation in the previous section. Apparently, D'_t is strictly decreasing with respect to t. Thus, we only need to guarantee $D_n \geq 0$.

By Gauss rounding function, the optimal number of sales stages denoted by n_o is obtained as follows:

$$n_o = G\left\langle \max n : \frac{a}{2} - \frac{\delta c}{2} - \frac{(n-1)a}{2T} \geq 0 \right\rangle. \tag{18}$$

According to (18), the selling cycle length is positively correlated to the shelf-life length. Moreover, the upper limit of the selling cycle length under the commission-charge mode is $(a/2 - \delta c/2)$.

Actually, (18) guarantees that D'_t meets the constraints of model (14) and model (15). By (18), we have

$$\frac{a}{2} - \frac{(n_o - 1)a}{2T} > 0,$$

i.e.,

$$n_o - 1 < T.$$

The relation between m_o and n_o will be revealed in the next section.

Similar to the previous section, the change rule of n_o is examined when a varies.

Following the above symbols, we assume that n_o is a critical point under some given a, i.e.,

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{(n_o - 1)a}{2T} = 0, \tag{19}$$

which suggests that the optimal number of sales stages definitely decreases when a reduces.

Consider the following inequality set:

$$\begin{cases} \dfrac{a - \Delta a}{2} - \dfrac{\delta c}{2} - \dfrac{(n_o - 1)(a - \Delta a)}{2T} < 0, \\ \dfrac{a - \Delta a}{2} - \dfrac{\delta c}{2} - \dfrac{(n_o - 2)(a - \Delta a)}{2T} \geq 0. \end{cases} \tag{20}$$

By (19), we transform (20) to the following form:

$$\begin{cases} \dfrac{(n_o - 1 - T)\Delta a}{2T} < 0, \\ \dfrac{(n_o - 2 - T)\Delta a + a}{2T} \geq 0. \end{cases}$$

By figuring out the above inequality set, we have

$$\Delta a \in \left(0, \frac{a}{T - n_o + 2}\right].$$

Namely, the optimal number of sales stages is equal to $n_o - 1$ when the demand decreases to $a - \Delta a$.

By analyzing the interval of Δa under the two sales modes, it is shown that the change rate of the number of sales stages entirely depends on the shelf-life length T and the market demand of a stage.

Next, we examine p'_t given by (16). According to (18), it is obtained that

$$p'_t - c = \frac{a}{2\delta} - \frac{c}{2} - \frac{(t-1)a}{2\delta T} \geq 0$$

holds for $t \leq n_o$. Hence, p'_t meets the constraints of model (14) and model (15).

For n_o given by (18), we have the following conclusion:

Proposition 2 In the commission-charge mode, the perishable product will be on sale during the whole shelf life as long as the following inequality holds:

$$\frac{a}{2T} - \frac{\delta c}{2} \geq 0.$$

Proof We use $n_o = T$ to represent the case that the perishable product will be on sale during the whole shelf life. Under

$$\frac{a}{2T} - \frac{\delta c}{2} \geq 0,$$

the discriminant function is transformed to

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{(n_o - 1)a}{2T} = \frac{Ta}{2T} - \frac{\delta c}{2} - \frac{(T-1)a}{2T} = \frac{a}{2T} - \frac{\delta c}{2} \geq 0.$$

According to the definition of the optimal number of sales stages, we have $n_o = T$, i.e., the perishable product will be on sale during the whole shelf life.

By Proposition 1 and Proposition 2, it is shown that perishable products in the commission-charge mode may possess a longer selling cycle length. We will conduct a detailed discussion in the following.

By (16) and (17), the profit of the whole supply chain is obtained as follows:

$$\begin{aligned} \pi'_2 = & \frac{a^2}{12\delta T^2} n_o^3 - \left(\frac{a^2}{8\delta T^2} + \frac{a^2}{4\delta T} - \frac{ac}{4T} \right) n_o^2 \\ & + \left(\frac{a^2}{24\delta T^2} + \frac{a^2}{4\delta T} - \frac{ac}{4T} + \frac{a^2}{4\delta} + \frac{\delta c^2}{4} - \frac{ac}{2} \right) n_o. \end{aligned} \tag{21}$$

Then $\pi'_{2,s}$ is obtained according to its definition:

$$\begin{aligned} \pi'_{2,s} = & (1-\beta) \frac{a^2}{12\delta T^2} n_o^3 - (1-\beta) \left(\frac{a^2}{8\delta T^2} + \frac{a^2}{4\delta T} - \frac{ac}{4T} \right) n_o^2 \\ & + (1-\beta) \left(\frac{a^2}{24\delta T^2} + \frac{a^2}{4\delta T} - \frac{ac}{4T} + \frac{a^2}{4\delta} + \frac{\delta c^2}{4} - \frac{ac}{2} \right) n_o. \end{aligned} \tag{22}$$

And $\pi'_{2,r}$ is

$$\begin{aligned} \pi'_{2,r} = & \beta \frac{a^2}{12\delta T^2} n_o^3 - \beta \left(\frac{a^2}{8\delta T^2} + \frac{a^2}{4\delta T} - \frac{ac}{4T} \right) n_o^2 \\ & + \beta \left(\frac{a^2}{24\delta T^2} + \frac{a^2}{4\delta T} - \frac{ac}{4T} + \frac{a^2}{4\delta} + \frac{\delta c^2}{4} - \frac{ac}{2} \right) n_o. \end{aligned} \tag{23}$$

Clearly, β is the decisive factor for profits of the supplier and the retailer. If β is too large, the profit of the supplier will be low, which means that the supplier may reject this sales mode. On the other hand, in order to guarantee that $\pi'_{2,r} \geq \pi^*_{1,r}$, it's impossible for the retailer to declare a low value of β. The reasonable interval of β will be presented in the following section.

6 Superiority Analysis

This section examines the superiority of each sales mode, and proposes an open interval of β on which the commission-charge mode is proved to be superior. In other words, the retailer will choose this sales mode and charge such a commission rate.

Firstly, we examine the sales prices at each stage for the two sales modes.

Apparently, (9) suggests that

$$\frac{a}{4} - \frac{\delta c}{4} - \frac{3(m_o - 1)a}{8T} \geq 0,$$

which leads to

$$\frac{a}{4} - \frac{\delta c}{4} - \frac{(m_o - 1)a}{8T} > 0$$

for $m_o \geq 2$.

By comparing the sales prices at stage$t$ for the two sales modes, we have

$$p'_t - p^*_t = -\frac{a}{4\delta} + \frac{c}{4} + \frac{(m_o - 1)a}{8\delta T} < 0.$$

From the above analysis, we come to the following proposition:

Proposition 3 When the number of sales stages for the traditional mode is greater than one, the sales price in the commission mode at each stage is definitely lower than the one in the traditional mode.

Similar to the above derivation, we also obtain that $p'_t \leq p^*_t$ when $m_o = 1$.

The above result reveals that consumers prefer to purchase perishable products from the commission-charge retailer in order to save expenditure.

Secondly, the numbers of sales stages of the two modes are compared.

We show the following proposition:

Proposition 4 When $m_o \geq 3$, $n_o \geq m_o + 1$.

Proof Clearly, (9) is equivalent to

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{3(m_o - 1)a}{4T} \geq 0. \tag{24}$$

We set $n_o = m_o + 1$, and substitute it into the discriminant given by (18):

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{m_o a}{2T}.$$

By $m_o \geq 3$ and (24), we have

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{m_o a}{2T} = \frac{a}{2} - \frac{\delta c}{2} - \frac{2m_o a}{4T} \geq \frac{a}{2} - \frac{\delta c}{2} - \frac{3(m_o - 1)a}{4T} \geq 0.$$

According to the definition of n_o (formula (18)), we have $n_o \geq m_o + 1$.

In practice, $n_o = m_o$ may occur when $m_o = 2$. We will illustrate this situation in the next section.

Further, we consider describing the relation between m_o and n_o precisely.

We have the following correlation between m_o and n_o:

Proposition 5 For any n_o,

$$G\left\langle \min m : m \geq \frac{2}{3}n_o - 1 \right\rangle \leq m_o < G\left\langle \max m : m < \frac{2}{3}n_o + 1 \right\rangle.$$

Proof On the premise that n_o is the optimal number of sales stages in the commission-charge mode, we have

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{(n_o - 1)a}{2T} \geq 0$$

according to (18).

We set

$$m_o = \frac{2}{3}n_o - 1,$$

and substitute it into the equivalent discriminant of (9):

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{3(m-1)a}{4T}.$$

Then we have

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{(2n_o - 6)a}{4T}.$$

It is obtained that

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{(2n_o - 6)a}{4T} > \frac{a}{2} - \frac{\delta c}{2} - \frac{(n_o - 1)a}{2T} \geq 0,$$

which implies that

$$m_o \geq \frac{2}{3}n_o - 1$$

according to the definition given by (9).

Next, we set

$$m_o = \frac{2}{3}n_o + 1,$$

and substitute it into the equivalent discriminant of (9). Then we have

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{2n_o a}{4T}.$$

For the above results, we have

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{2n_o a}{4T} < \frac{a}{2} - \frac{\delta c}{2} - \frac{(n_o - 1)a}{2T}.$$

However, it is not enough to determine the upper bound of m_o.

According to (18), when the number of sales stages in the commission-charge mode is $n_o + 1$, we have

$$\frac{a}{2} - \frac{\delta c}{2} - \frac{n_o a}{2T} < 0.$$

Hence, according to the definition given by (9), we have

$$m_o < \frac{2}{3}n_o + 1.$$

Given the above, the relation between m_o and n_o is achieved as follows:

$$\frac{2}{3}n_o - 1 \leq m_o < \frac{2}{3}n_o + 1.$$

Besides, $m_o \in Z^+$ and $n_o \in Z^+$ according to the definition. Hence, the equivalent form for the above inequalities is obtained as follows:

$$G\left\langle \min m : m \geq \frac{2}{3}n_o - 1 \right\rangle \leq m_o < G\left\langle \max m : m < \frac{2}{3}n_o + 1 \right\rangle.$$

Apparently, in proposition 5, the description of the relation between m_o and n_o is not explicit enough. Similar to the given notation, we denote by $\langle x \rangle$ the maximum integer not exceeding x. Thus, we present an equivalent statement for proposition 5 as follows:

Corollary 1 $\left\langle \frac{2}{3}n_o - \frac{1}{k} \right\rangle \leq m_o < \left\langle \frac{2}{3}n_o - \frac{1}{k} \right\rangle + 1$ for any $k > 3$.

Proof Actually, $2n_o/3$ can be expressed as

$$\frac{2}{3}n_o = i + \frac{j}{3},$$

in which i and j are both integers and $0 \leq j < 3$. When $j = 0$, we have

$$\left\langle i - \frac{1}{k} \right\rangle = i - 1 = G\left\langle \min m : m \geq i - 1 \right\rangle.$$

On the other hand, when $j = 1$ or 2, we have

$$\left\langle i + \frac{j}{3} - \frac{1}{k} \right\rangle = i = G\left\langle \min m : m \geq i + \frac{j}{3} - 1 \right\rangle.$$

Hence, $\left\langle \frac{2}{3}n_o - \frac{1}{k} \right\rangle = G\left\langle \min m : m \geq \frac{2}{3}n_o - 1 \right\rangle.$

By similar analysis, we can also obtain

$$\left\langle \frac{2}{3}n_o - \frac{1}{k} \right\rangle + 1 = G\left\langle \max m : m < \frac{2}{3}n_o + 1 \right\rangle.$$

Corollary 1 shows the explicit relation between m_o and n_o. In practice, the form given by corollary 1 may be more convenient to be applied.

It is shown by Proposition 4 and Proposition 5 that the selling cycle length in the commission-charge mode is always longer than the one in the traditional mode. By associating with Proposition 3, consumers can acquire perishable products at a lower price in the commission mode if they pay little attention to the freshness.

Finally, we compare the profits of each participant under different sales modes.

We assume that there exists an open interval $\Omega \subset [0, 1]$: When $\beta \in \Omega$, we have

$$\pi'_{2,s} = (1 - \beta)\pi'_2 > \pi^*_{1,s}$$

and

$$\pi'_{2,r} = \beta\pi'_2 > \pi^*_{1,r}.$$

By solving the above inequalities, we have

$$\frac{\pi^*_{1,r}}{\pi'_2} < \beta < 1 - \frac{\pi^*_{1,s}}{\pi'_2}. \tag{25}$$

The region Ω is then given by (25):

$$\Omega = \left(\frac{\pi_{1,r}^*}{\pi_2'}, 1 - \frac{\pi_{1,s}^*}{\pi_2'}\right).$$

According to Lou, et al.[25], the total profit of a two-layer perishable-product supply chain in the centralized decision-making case is higher than the one in the decentralized case. Thus, we have

$$\pi_2' > \pi_1^* = \pi_{1,s}^* + \pi_{1,r}^*.$$

Hence

$$1 - \frac{\pi_{1,s}^*}{\pi_2'} - \frac{\pi_{1,r}^*}{\pi_2'} = \frac{\pi_2' - \pi_{1,s}^* - \pi_{1,r}^*}{\pi_2'} > 0,$$

Ω is not an empty set.

Apparently, both the supplier and the retailer gain more profits when the retailer chooses the commission mode and the commission rate $\beta \in \Omega$. In addition, consumers purchase the same product by paying less expenditures. Hence, this situation is to the satisfaction of all.

Nevertheless, no specific commission rate is given by the above discussion. Actually, more factors may be taken into consideration to figure out an optimal commission rate. Here we present an ideal value for the commission rate, under which both the supplier and the retailer achieve an equal proportion raise on their revenue comparing with the ones in the traditional sales mode.

In order to guarantee that the two participants gain an equal relative increment, we consider the following equation:

$$\frac{{\pi_{2,s}}' - {\pi_{1,s}}^*}{{\pi_{1,s}}^*} = \frac{{\pi_{2,r}}' - {\pi_{1,r}}^*}{{\pi_{1,r}}^*}.$$

By substituting π_2' and β, we have

$$\frac{(1-\beta){\pi_2}' - {\pi_{1,s}}^*}{{\pi_{1,s}}^*} = \frac{\beta{\pi_2}' - {\pi_{1,r}}^*}{{\pi_{1,r}}^*}.$$

By solving the above equation, we have

$$\beta = \frac{\pi_{1,r}^*}{\pi_{1,s}^* + \pi_{1,r}^*}. \tag{26}$$

Finally, we verify if the value of β determined by (26) locates in Ω. Because

$$\pi_2' > \pi_1^* = {\pi_{1,s}}^* + {\pi_{1,r}}^*.$$

We have

$$\frac{{\pi_{1,r}}^*}{{\pi_{1,s}}^* + {\pi_{1,r}}^*} - \frac{{\pi_{1,r}}^*}{{\pi_2}'} > 0$$

and

$$1 - \frac{{\pi_{1,s}}^*}{{\pi_2}'} - \frac{{\pi_{1,r}}^*}{{\pi_{1,s}}^* + {\pi_{1,r}}^*} > 1 - \frac{{\pi_{1,s}}^*}{{\pi_{1,s}}^* + {\pi_{1,r}}^*} - \frac{{\pi_{1,r}}^*}{{\pi_{1,s}}^* + {\pi_{1,r}}^*} = 0.$$

Hence, the value of β given by (26) locates in Ω.

7 A Numerical Illustration

In order to visualize the proposed models and verify the obtained conclusions under different market demands of each isometric stage, we present a numerical example.

First, we analyze the change of the selling cycle length under each sales mode. Consider the following scenario: $a \in [300, 1000]$, $c = 100$, $\delta = 2$, $T = 5$.

According to (9), we have

$$m_o = G\left\langle \max m : \frac{a}{4} - 50 - \frac{3(m-1)a}{40} \geq 0 \right\rangle.$$

According to (18), we have

$$n_o = G\left\langle \max n : \frac{a}{2} - 100 - \frac{(n-1)a}{10} \geq 0 \right\rangle.$$

Then the changing curves of the two variables are depicted by the following graph:

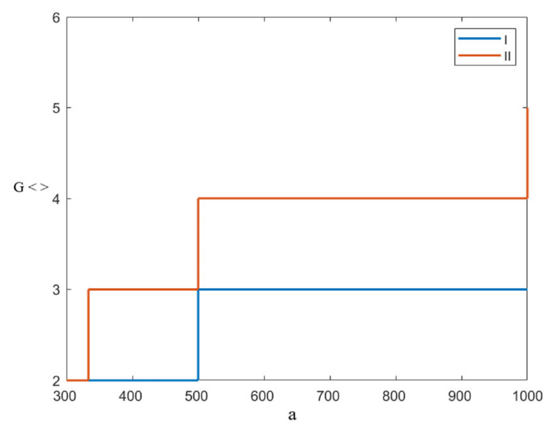

Figure 1 Changes of selling cycle lengths

In Figure 1, I represents the selling cycle length of the traditional sales mode, and II represents the selling cycle length of the commission sales mode.

As it is shown, the selling cycle lengths of the two modes may be equal when $m_o = 2$, which makes a supplement for Proposition 4.

Moreover, when $a = 1000$, the selling cycle length of the commission mode is equal to $T = 5$. Because

$$\frac{a}{2T} - \frac{\delta c}{2} = 0,$$

this result coincides with Proposition 2.

Next, we turn to determine the open interval Ω.

According to the analysis, the lower bound of β meets the following equality constraint:

$$\beta\left[\frac{a^2}{600}n_o^3 - \left(\frac{11a^2}{400} - 5a\right)n_o^2 + \left(\frac{181a^2}{1200} - 55a + 5000\right)n_o\right]$$
$$= \frac{7a^2}{9600}m_o^3 - \left(\frac{11a^2}{1600} - \frac{5a}{4}\right)m_o^2 + \left(\frac{359a^2}{9600} - \frac{55a}{4} + 1250\right)m_o.$$

The upper bound of β meets the following equality constraint:

$$(1-\beta)\left[\frac{a^2}{600}n_o^3-\left(\frac{11a^2}{400}-5a\right)n_o^2+\left(\frac{181a^2}{1200}-55a+5000\right)n_o\right]$$
$$=\frac{a^2}{1600}m_o^3-\left(\frac{11a^2}{800}-\frac{5a}{2}\right)m_o^2+\frac{(11a-2000)^2}{1600}m_o.$$

And the function of the ideal value of β is given as follows:

$$\frac{\frac{7a^2}{9600}m^3-(\frac{11a^2}{1600}-\frac{5a}{4})m^2+(\frac{359a^2}{9600}-\frac{55a}{4}+1250)m}{[\frac{7a^2}{9600}m^3-(\frac{11a^2}{1600}-\frac{5a}{4})m^2+(\frac{359a^2}{9600}-\frac{55a}{4}+1250)m]+[\frac{a^2}{1600}m^3-(\frac{11a^2}{800}-\frac{5a}{2})m^2+\frac{(11a-2000)^2}{1600}m]}$$

In the above equalities, m_o is determined by the following function:

$$m_o=G\left\langle\max m:\frac{a}{4}-50-\frac{3(m-1)a}{40}\geq 0\right\rangle.$$

And n_o is determined by the following function:

$$n_o=G\left\langle\max n:\frac{a}{2}-100-\frac{(n-1)a}{10}\geq 0\right\rangle.$$

Given the above, the interval Ω comprised by the lower bound and the upper bound of β under different a is depicted as follows:

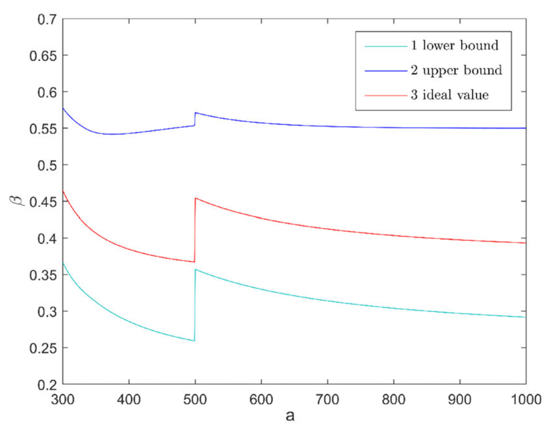

Figure 2 The lower bound and the upper bound of β under different a modes

In Figure 2, curve 1 represents the lower bound of β, curve 2 represents the upper bound of β, and curve 3 represents the ideal value of β.

For any value of a, the interval Ω is determined by the lower bound and the upper bound of β. As is shown by Figure 2, the ideal value of β which guarantees that the two participants gain an equal relative increment always locates in Ω.

Finally, we examine the impact of the shelf-life length over the profit under different sales modes. We consider the following scenario: $a=800$, $c=100$, $\delta=2$, $T\in[5,10]$.

According to (9), the selling cycle length under traditional mode is

$$m_o=G\left\langle\max m:150-\frac{300(m-1)}{T}\geq 0\right\rangle.$$

According to (18), the selling cycle length under commission-charge mode is

$$n_o = G\left\langle \max n : 300 - \frac{400(n-1)}{T} \geq 0 \right\rangle .$$

The profit of the traditional mode is $\pi_1^* = \pi_{1,s}^* + \pi_{1,r}^*$, where $\pi_{1,s}^*$ and $\pi_{1,r}^*$ are given by (12) and (13) respectively. And the profit of the commission-charge mode is π_2', which is given by (21).

According to the above results, the two curves are presented as follows:

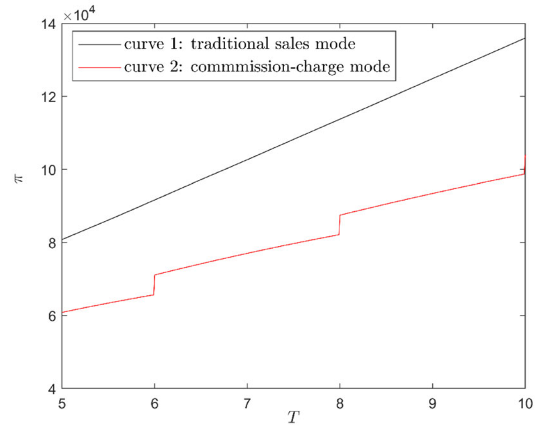

Figure 3 Profit curves of different modes

As is shown by Figure 3, the profit of the commission-charge mode is always larger than the profit of the traditional mode, which coincides with our conclusion in superiority analysis. Besides, the profit function of the commission-charge mode is more sensitive to the shelf-life length.

8 Conclusions

In this research, sales mode selection issues for a two-echelon supply chain of perishable products are considered. The retailer is the one who determines the type of the sales mode. The superiorities of two different sales modes are analyzed. Actually, in the commission-charge mode, the supplier conducts a direct selling, and then shares the profits with the retailer who is in charge of the practical sales.

Our research draws some significant conclusions, which cover some research gaps for existing literature. First, it is demonstrated from the theoretical perspective that the selling cycle length in the traditional sales mode often cannot be as long as the shelf-life length of the product. Second, we show that the sales price at each stage is lower and meanwhile the selling cycle length is longer in the commission-charge mode. Third, a non-empty interval of the commission rate is proposed, on which the commission-charge mode is to the satisfaction of all.

Some managerial insights are revealed. As is shown in this paper, the commission-charge mode is more superior than the traditional-charge mode when selling perishable products, not only for the supplier and the retailer, but also for consumers. In the traditional sales mode, the retailer has little motivation to order many perishable products so as to avoid a long selling cycle length. In the commission-charge mode, the supplier should prepare enough products to guarantee that no shortage happens during the whole selling cycle length when the inequality in Proposition 2 holds.

In our future research, ordering and pricing strategies for perishable products with different production dates will be explored, as a supplement for this study. In practice, it is common for a supermarket to sell perishable products with different production dates. Furthermore, as is shown by our numerical illustrations, an extension on the shelf-life can raise the profit of the supply chain. In the US grocery industry, approximately 30 billion is lost annually because of the deterioration of fresh produce and foods (Chen, et al.[27]). Hence, the investment in preservation technology to improve the freshness of perishable products is also an important research direction (Ma, et al.[28]).

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