

# Maclaurin Symmetric Mean Aggregation Operators and Their Application to Hesitant Q-Rung Orthopair Fuzzy Multiple Attribute Decision Making

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**Abstract** The Maclaurin symmetric mean (MSM) operator exhibits a desirable characteristic by effectively capturing the correlations among multiple input parameters, and it serves as an extension of certain existing aggregation operators through adjustments to the parameter  $k$ . The hesitant q-rung orthopair set (Hq-ROFSs) can serve as an extension of the existing orthopair fuzzy sets, which provides decision makers more freedom in describing their true opinions. The objective of this paper is to present an MSM operator to aggregate hesitant q-rung orthopair numbers and solve the multiple

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attribute decision making (MADM) problems in which the attribute values take the form of hesitant  $q$ -rung orthopair fuzzy sets (H- $q$ ROFSs). Firstly, the definition of H- $q$ ROFSs and some operational laws of H- $q$ ROFSs are proposed. Then we develop a family of hesitant  $q$ -rung orthopair fuzzy maclaurin symmetric mean aggregation operators, such as the hesitant  $q$ -rung orthopair fuzzy maclaurin symmetric mean (Hq-ROFMSM) operator, the hesitant  $q$ -rung orthopair fuzzy weighted maclaurin symmetric mean (Hq-ROFWMSM) operator, the hesitant  $q$ -rung orthopair fuzzy dual maclaurin symmetric mean (Hq-ROFDMSM) operator, the hesitant  $q$ -rung orthopair fuzzy weighted dual maclaurin symmetric mean (Hq-ROFWDMSM) operator. And the properties and special cases of these proposed operators are studied. Furthermore, an approach based on the Hq-ROFWMSM operator is proposed for multiple attribute decision making problems under hesitant  $q$ -rung orthopair fuzzy environment. Finally, a numerical example and comparative analysis is given to illustrate the application of the proposed approach.

**Keywords** hesitant  $q$ -rung orthopair fuzzy sets; multiple attribute decision making; Maclaurin symmetric mean operator; hesitant  $q$ -rung orthopair fuzzy weighted dual MSM (Hq-ROFWDMSM) operator; hesitant  $q$ -rung orthopair fuzzy weighted MSM (Hq-ROFWMSM) operator

## 1 Introduction

Decision-making problems widely exist in social development. Whether it is residents investing in housing, purchasing goods and tourism, or account investment risk assessment, performance appraisal and other daily activities, scientific and comprehensive decision-making is required. With the increasingly complex decision-making environment and the fuzziness of decision-making experts' cognitive thinking, fuzzy decision-making based on Fuzzy set<sup>[1]</sup> has been widely studied<sup>[2-7]</sup>. With the continuous deepening of human research on information fuzziness in the real world, some scholars have enriched and improved the theory of Fuzzy set, and put forward theories such as interval valued Fuzzy set (IVFS)<sup>[8]</sup>, type 2 Fuzzy set (T2FS)<sup>[9]</sup>, and type  $n$  fuzzy set (TnFS)<sup>[10]</sup>. However, this kind of Fuzzy set can only describe the degree that an element belongs to a set, and is not perfect in describing uncertain phenomena. So Atanassov<sup>[11]</sup> proposed intuitionistic Fuzzy set (IFS) in 1983. It contains two dimensions at the same time, namely "membership" and "non membership". Among them, membership degree represents the degree to which an element belongs to a certain set, while non membership degree represents the degree to which the element does not belong to a certain set. It is worth noting that the constraint that IFS need to meet is that the sum of membership and non membership is less than or equal to 1. However, such constraints may not be strictly met in the real world. Yager noticed that in practical decision-making problems, it is common to encounter situations where the sum of membership and non membership is greater than 1, and the sum of the squares of the two is less than 1<sup>[12]</sup>.

Obviously, IFS are not suitable for describing such uncertain phenomena. So Yager<sup>[13]</sup> extended the theory of IFS and proposed Pythagoras Fuzzy set (PFS). The constraint condition of PFS is that the sum of the squares of membership and non membership is less than or equal to 1. Obviously, the constraints of PFS are broader than those of IFS, so PFS can bring less information distortion and are more suitable for describing uncertainty. Because of this characteristic of PFS, the multi-attribute decision-making problem based on PFS has become a new research hotspot. Later, Yager<sup>[14]</sup> further popularized PFS and proposed a generalized

form of Pythagoras Fuzzy set, namely  $q$ -rung orthopair fuzzy sets ( $q$ -ROFSs). The constraint condition that the  $q$ -ROFSs needs to satisfy is that the sum of the  $q$  power of the membership degree and the  $q$  power of the non membership degree is less than or equal to 1. It should be pointed out that the so-called  $q$ -ROFSs refers to a Fuzzy set that contains both membership and non membership dimensions, so IFS, PFS and  $q$ -ROFSs all belong to orthopair fuzzy sets. According to the constraints of  $q$ -ROFSs, IFS and PFS are special cases of  $q$ -ROFSs. When  $q = 1$ ,  $q$ -ROFSs degenerate into IFS. When  $q = 2$ ,  $q$ -ROFSs degenerate into PFS. As a new generalized type of fuzzy sets,  $q$ -ROFS has received a considerable attention and has been applied to a various field of decision making<sup>[15–18]</sup>. In routine matters, individuals frequently display hesitation and a lack of definitiveness when confronted with decision-making tasks, ultimately leading to challenges for the decision-maker in arriving at a conclusion. In response to these challenges, Torra<sup>[19]</sup> introduced the concept of the “hesitant fuzzy set” (HFS) as a means to tackle such indecisiveness. In this theory, instead of a single numerical value, the degree of membership for each element of the reference set is represented by a set of discrete values within the interval  $[0, 1]$ . Further, Hussain, et al.<sup>[20]</sup> introduced hesitant  $q$ -ROPFS (Hq-ROPFS) as the hybrid structure of hesitant and IFS, PFS,  $q$ -ROPFS, respectively.

Qin, et al.<sup>[21]</sup> discussed a novel choquet integral-based VIKOR approach under  $q$ -rung orthopair hesitant fuzzy environment. Kavitha, et al.<sup>[22]</sup> studied multi-label feature selection using  $q$ -rung orthopair hesitant fuzzy MCDM approach extended to CODAS. Farid, et al.<sup>[23]</sup> studied optimizing filtration technology for contamination control in gas processing plants using hesitant  $q$ -rung orthopair fuzzy information aggregation. Zhang and Yuan<sup>[24]</sup> developed generalized interval-valued  $q$ -rung orthopair hesitant fuzzy choquet operators. Muhammad, et al.<sup>[25]</sup> studied dual hesitant  $q$ -rung orthopair fuzzy Hamacher graphs with application. Zhang, et al.<sup>[26]</sup> proposed dual hesitant  $q$ -rung orthopair fuzzy interaction partitioned Bonferroni Mean operators. Ashraf, et al.<sup>[27]</sup> proposed a decision making algorithm for wind power plant based on  $q$ -rung orthopair hesitant fuzzy rough aggregation information and TOPSIS. Chen, et al.<sup>[28]</sup> developed multi-attribute decision-making method based on  $q$ -rung orthopair probabilistic hesitant fuzzy schweizer-sklar power weighted hamy mean operator. Liu, et al.<sup>[29]</sup> discussed the cross-entropy and improved distance measures for complex  $q$ -rung orthopair hesitant fuzzy sets and their applications in multi-criteria decision-making.

The MSM operator was originally introduced by Maclaurin<sup>[30]</sup> and then developed by Detemple and Robertson<sup>[31]</sup>. The prominent characteristic of the MSM is that it can capture the interrelationship among the multi-input arguments. The MSM operator can provide more flexible and robust information fusion and make it more adequate to solve MADM in which the attributes are independent. Moreover, for a given collection of arguments, the MSM operator is monotonically decreasing with respect to the values of the parameter, which can reflect the risk preferences of the decision makers in practical situations. In the past few years, the MSM has received more and more attention, and many important results both in theory and application are developed. Garg, et al.<sup>[32]</sup> studied algorithm for multiple attribute decision-making using T-spherical fuzzy Maclaurin symmetric mean operator. Yang, et al.<sup>[33]</sup> proposed neutrosophic cubic Maclaurin symmetric mean aggregation operators with applications in multi-criteria decision-making. Chen and Ye<sup>[34]</sup> developed a novel MAGDM Method

based on hesitant picture fuzzy schweizer-sklar maclaurin symmetric mean operators and their application. Tahir and Ubaid<sup>[35]</sup> proposed multi-attribute decision-making method based on bipolar complex fuzzy Maclaurin symmetric mean operators. Chen and Zhang<sup>[36]</sup> proposed pythagorean fuzzy linguistic power generalized Maclaurin symmetric mean based on novel operations and their application in multiple attribute group decision-making. Ning, et al.<sup>[37]</sup> discussed a novel MADM technique based on extended power generalized Maclaurin symmetric mean operators under probabilistic dual hesitant fuzzy setting and its application to sustainable suppliers selection. Ali, et al.<sup>[38]</sup> proposed complex intuitionistic fuzzy Maclaurin, symmetric mean operators and its application to emergency program selection. Song and Geng<sup>[39]</sup> proposed some single-valued neutrosophic uncertain linguistic maclaurin symmetric mean operators and their application to multiple-attribute decision making. Aliya, et al.<sup>[40]</sup> proposed maclaurin symmetric mean aggregation operators based on cubic Pythagorean linguistic fuzzy number. Mu, et al.<sup>[41]</sup> proposed a novel approach to multi-attribute group decision-making based on interval-valued Pythagorean fuzzy power Maclaurin symmetric mean operator. Liu and Zhang<sup>[42]</sup> proposed multiple-attribute decision making method based on power generalized maclaurin symmetric mean operators under normal wiggly hesitant fuzzy environment. Yang, et al.<sup>[43]</sup> studied three-way decisions based on q-rung orthopair fuzzy 2-tuple linguistic sets with generalized maclaurin symmetric mean operators.

From the above analysis, we can see that most of the existing aggregation operators are based on the algebraic product and algebraic sum to carry the aggregation process that does not consider the interrelationship among the multi-input arguments. In recent years, many researchers studied the MSM operator and its applications that only consider the case that arguments take the form of crisp numbers. And it seems that there is no investigation on how to combine the maclaurin symmetric mean operators with hesitant q-rung orthopair fuzzy information. To do so, the remainder of this paper is set out as follows. In Section 2, we introduce some basic concepts related to q-rung orthopair fuzzy sets ( $q$ -ROFSs) and some operational laws of hesitant q-rung orthopair fuzzy (Hq-ROFSs). In Section 3, we have developed some hesitant q-rung orthopair fuzzy maclaurin symmetric mean operator's aggregation operators and studied some desirable properties of the proposed operators. In Section 4, we have applied these operators to develop some models for multiple attribute decision making (MADM) problems based on the maclaurin symmetric mean operator's aggregation operators with hesitant q-rung orthopair fuzzy information. In Section 5, a practical example is given to verify the developed approach and to demonstrate its practicality and effectiveness. In Section 6, we conclude the paper and give some remarks.

## 2 Preliminaries

### 2.1 Hesitant Fuzzy Set

**Definition 1**<sup>[19]</sup> Let  $X$  be a reference set, an HFS  $A$  on  $X$  is denoted by a function  $h_A(x)$  that returns a subset of  $[0, 1]$  when it is applied to  $X$ . To make it easier to understood, the HFS is expressed by a mathematical symbol

$$A = \{ \langle x, h_{A(x)} \rangle \mid x \in X \}, \quad (1)$$

where  $h_{A(x)}$  is a set of some different values indicating all possible membership degrees of the element  $x \in X$  to the set  $A$ . For convenience,  $h_{A(x)}$  is called a hesitant fuzzy element (HFE), which is denoted by  $h_{A(x)} = \{\gamma \mid \gamma \in h_{A(x)}\}$ .

Let  $h, h_1$  and  $h_2$  be three HFEs, then some operations are presented as follows:

- 1)  $h_1 \oplus h_2 = \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2 \mid \gamma_1 \in h_1, \gamma_2 \in h_2\}$ .
- 2)  $h_1 \otimes h_2 = \{\gamma_1\gamma_2 \mid \gamma_1 \in h_1, \gamma_2 \in h_2\}$ .
- 3)  $\lambda h = \{1 - (1 - \gamma)^\lambda \mid \gamma \in h\}$ .
- 4)  $h^\lambda = \{(\gamma)^\lambda \mid \gamma \in h\}$ .
- 5)  $h^c = \{1 - \gamma \mid \gamma \in h\}$ .

**Definition 2**<sup>[19]</sup> For an HFE  $h = \{\gamma \mid \gamma \in h\}$ ,  $S(h) = \frac{1}{l_h} \sum_{\gamma \in h} \gamma$  is called the score function of  $h$ , where  $l_h$  is the number of the values in  $h$ . For two HFEs  $h_1$  and  $h_2$ , if  $S(h_1) > S(h_2)$ , then  $h_1 > h_2$ ; if  $S(h_1) = S(h_2)$ , then  $h_1 = h_2$ .

## 2.2 Q-rung Fuzzy Set

Yager<sup>[14]</sup> originally proposed the concept of q-ROFSs, which is the generalization of the classical IFSs and PFSs.

**Definition 3**<sup>[14]</sup> Let  $X$  be a fix set. A Q-ROFSs is an object having the form

$$P = \left\{ \left\langle x, (u_p(x), v_p(x))_q \right\rangle \mid x \in X \right\}, \quad (2)$$

where the function  $u_p : X \rightarrow [0, 1]$  defines the degree of membership and the function  $v_p : X \rightarrow [0, 1]$  defines the degree of nonmembership of the element  $x \in X$  to  $P$ , respectively, and for every  $x \in X$ , it holds that

$$(u_p(x)^q + v_p(x)^q) \leq 1, \quad q \geq 1.$$

The degree of indeterminacy is given as  $\pi_p(x) = \sqrt[q]{1 - (u_p(x))^q - (v_p(x))^q}$ .

For convenience, we called  $p = (u, v)$  a  $q$ -rung orthopair fuzzy element (Q-ROFE).

**Definition 4** Let  $p = (u, v)$  be a Q-ROFE, then a score function can be represented as follows:

$$S(p) = \frac{1}{2} (1 + u^q - v^q), \quad S(a) \in [0, 1]. \quad (3)$$

**Definition 5** Let  $p = (u, v)$  be a Q-ROFE, an accuracy function can be represented as follows:

$$H(p) = u^q + v^q, \quad H(a) \in [0, 1] \quad (4)$$

to evaluate the degree of accuracy of the Q-ROFE  $p = (u, v)$ . The larger the value of  $H(p)$ , the more the degree of accuracy of the q-ROFE  $p$  is.

Based on the score function  $S$  and the accuracy function  $H$ , in the following, Liu, et al.<sup>[5]</sup> proposed a comparison between two q-ROFEs, which is defined as follows:

**Definition 6** Let  $p_1, p_2$  be two q-ROFEs. Then,

- 1) If  $S(p_1) > S(p_2)$ , then  $p_1 > p_2$ .
- 2) If  $S(p_1) = S(p_2)$ , then
  - If  $H(p_1) > H(p_2)$ , then  $p_1 > p_2$ .
  - If  $H(p_1) = H(p_2)$ , then  $p_1 = p_2$ .

### 2.3 Hesitant Q-rung Orthopair Fuzzy Sets

In the following, on the basic of hesitant fuzzy set and q-rung orthopair fuzzy set, the concept of the hesitant q-rung orthopair fuzzy set (Hq-ROFS) was proposed<sup>[44]</sup>.

**Definition 7**<sup>[20]</sup> Given a fixed set  $X$ , a hesitant q-rung orthopair fuzzy set (Hq-ROFS) on  $X$  is in terms of a function that when applied to  $X$  returns a subset of  $[0, 1]$ . The hesitant q-rung orthopair fuzzy set can be expressed by mathematical symbol:

$$P = \left\{ \left\langle x, \langle g_{P(x)}, h_{P(x)} \rangle_q \right\rangle \mid x \in X \right\}, \quad (5)$$

where  $g_{P(x)}, h_{P(x)}$  is a set of some values in  $[0, 1]$ , denoting the possible membership degree and the possible nonmembership degree of the element  $x \in X$  to the set  $P$ , respectively, and for every  $x \in X$ , it holds that

$$(g_p(x))^q + h_p(x)^q \leq 1, \quad q \geq 1.$$

For convenience, we called  $p = \langle \{u\}, \{v\} \mid u \in g, v \in h \rangle$  a hesitant q-rung orthopair fuzzy element (Hq-ROFE).

**Definition 8** Let  $p = \langle g, h \rangle, p_1 = \langle g_1, h_1 \rangle$  and  $p_2 = \langle g_2, h_2 \rangle$  be three hesitant q-rung orthopair fuzzy elements, and some basic operations on them are defined as follows:

- 1)  $p_1 \oplus p_2 = \left\langle \left\{ \sqrt[q]{u_1^q + u_2^q - u_1^q u_2^q} \right\}, \{v_1 v_2\} \right\rangle$ .
- 2)  $p_1 \otimes p_2 = \left\langle \{v_1 v_2\}, \left\{ \sqrt[q]{u_1^q + u_2^q - u_1^q u_2^q} \right\} \right\rangle$ .
- 3)  $\lambda p = \left\langle \left\{ \sqrt[q]{1 - (1 - u^q)^\lambda} \right\}, \{v^\lambda\} \right\rangle, \lambda > 0$ .
- 4)  $p^\lambda = \left\langle \{u^\lambda\}, \left\{ \sqrt[q]{1 - (1 - u^q)^\lambda} \right\} \right\rangle, \lambda > 0$ .
- 5)  $p^c = \langle \{v\}, \{u\} \rangle$ .

**Definition 9** Let  $p = \langle g, h \rangle$  be an Hq-ROFE, then we define the score function of  $p$  as follows:

$$S(p) = \left( \frac{1}{l_{u \in g}} \sum_{u \in g} u \right)^q - \left( \frac{1}{l_{v \in h}} \sum_{v \in h} v \right)^q, \quad (6)$$

where  $S(p) \in [-1, 1]$ ,  $l_u$  denotes the number of elements in  $g$ , and  $l_v$  denotes the number of elements in  $h$ .

**Definition 10** Let  $p = \langle g, h \rangle$  be an Hq-ROFE, then we define the accuracy function of  $p$  as follows:

$$H(p) = \left( \frac{1}{l_{u \in g}} \sum_{u \in g} u - S(p) \right)^q - \left( \frac{1}{l_{v \in h}} \sum_{v \in h} v - S(p) \right)^q. \quad (7)$$

Here we can see that  $S(p)$  is just the mean value in statistics, and  $H(p)$  is just the standard variance, which reflects the accuracy degree between all values in the Hq-ROFE and their mean value. Inspired by this idea, based on the score  $S(p)$  and the accuracy degree  $H(p)$ , we can compare and rank two Hq-ROFEs in the following definition.

**Definition 11** Let  $p_1 = \langle g_1, h_1 \rangle$  and  $p_2 = \langle g_2, h_2 \rangle$  be two Hq-ROFEs,  $S(p_1)$  be the score function of  $p_1$  and  $S(p_2)$  be the score function of  $p_2$ ,  $H(p_1)$  be the accuracy function of  $p_1$  and  $H(p_2)$  be the accuracy function of  $p_2$ . Then

- 1) If  $S(p_1) < S(p_2)$ , then  $p_1 < p_2$ .

- 2) If  $S(p_1) > S(p_2)$ , then  $p_1 > p_2$ .
- 3) If  $S(p_1) = S(p_2)$ , then  $p_1 \sim p_2$ .
  - If  $H(p_1) < H(p_2)$ , then  $p_1 < p_2$ .
  - If  $H(p_1) > H(p_2)$ , then  $p_1 > p_2$ .
  - If  $H(p_1) = H(p_2)$ , then  $p_1 = p_2$ .

**Definition 12** Let  $p_1 = \langle g_1, h_1 \rangle$  and  $p_2 = \langle g_2, h_2 \rangle$  be two Hq-ROFEs on a set  $X = \{x_1, x_2, \dots, x_n\}$ , then we define the distance measure between  $p_1$  and  $p_2$  as follows:

$$D(p_1, p_2) = \frac{1}{q} \left[ \frac{1}{l_{x_i}} \sum_{i=1}^{l_{x_i}} \left| \left( g_{p_1}^{\sigma(k)}(x_i) \right)^q - \left( g_{p_2}^{\sigma(k)}(x_i) \right)^q \right| + \frac{1}{l_{x_i}} \sum_{i=1}^{l_{x_i}} \left| \left( h_{p_1}^{\sigma(k)}(x_i) \right)^q - \left( h_{p_2}^{\sigma(k)}(x_i) \right)^q \right| \right],$$

where  $g_{p_1}^{\sigma(k)}(x_i), g_{p_2}^{\sigma(k)}(x_i), h_{p_1}^{\sigma(k)}(x_i), h_{p_2}^{\sigma(k)}(x_i)$  be the  $i^{\text{th}}$  largest value in  $p_1$  and  $p_2$  respectively.

## 2.4 MSM Operator

The MSM operator was originally introduced in [30, 31], which is a useful technique characterized by the ability to capture the interrelationship among the multi-input arguments. The definition of MSM is defined as follows.

**Definition 13** Let  $a_j$  ( $j = 1, 2, \dots, n$ ) be a collection of nonnegative numbers, and  $k = 1, 2, \dots, n$ . If

$$\text{MSM}^{(k)}(a_1, a_2, \dots, a_n) = \left( \frac{\sum_{1 \leq i_1 \leq \dots \leq i_k \leq n} \prod_{j=1}^k a_{i_j}}{C_n^k} \right)^{\frac{1}{k}}, \quad (8)$$

then  $\text{MSM}^{(k)}$  is called the MSM operator, where  $(i_1, i_2, \dots, i_k)$  traverse all the  $k$ -tuple combinations of  $(1, 2, \dots, n)$ , and  $C_n^k$  is the binomial coefficient.

Obviously, the MSM has the following properties:

$$\text{MSM}^{(k)}(0, 0, \dots, 0) = 0;$$

$$\text{MSM}^{(k)}(a, a, \dots, a) = a;$$

$$\text{MSM}^{(k)}(a_1, a_2, \dots, a_n) \leq \text{MSM}^{(k)}(b_1, b_2, \dots, b_n), \text{ if } a_i \leq b_i \text{ for all } i;$$

$$\min_i \{a_i\} \leq \text{MSM}^{(k)}(a_1, a_2, \dots, a_n) \leq \max_i \{a_i\}.$$

## 2.5 DMSM Operator

Qin and Liu<sup>[45]</sup> proposed the dual MSM (DMSM) operator considering both the MSM and the dual operation.

**Definition 14** Let  $a_j$  ( $j = 1, 2, \dots, n$ ) be a collection of nonnegative numbers, and  $k = 1, 2, \dots, n$ . If

$$\text{DMSM}^{(k)}(a_1, a_2, \dots, a_n) = \frac{1}{k} \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( \sum_{j=1}^k a_{i_j} \right)^{\frac{1}{C_n^k}} \right), \quad (9)$$

then  $\text{DMSM}^{(k)}$  is called the DMSM operator, where  $(i_1, i_2, \dots, i_k)$  traverse all the  $k$ -tuple combinations of  $(1, 2, \dots, n)$ , and  $C_n^k$  is the binomial coefficient.

### 3 Hesitant Q-rung Orthopair Fuzzy MSM Aggregation Operators

#### 3.1 Hesitant Q-rung Orthopair Fuzzy MSM Aggregation Operators

**Definition 15** Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, the Hq-ROFMSM operator is a mapping Hq-ROFMSM:

Hq-ROFE  $^n \rightarrow$  Hq-ROFE can be defined as

$$Hq\text{-ROFMSM } M^{(k)}(p_1, p_2, \dots, p_n) = \left( \frac{\bigoplus_{1 \leq i_1 \leq \dots \leq i_k \leq n} (\otimes_{j=1}^k p_{i_j})}{C_n^k} \right)^{\frac{1}{k}}. \quad (10)$$

**Theorem 1** Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, then the aggregation result using Hq-ROFMSM operator is also an Hq-ROFE and

$$\begin{aligned} & Hq\text{-ROFMSM } M^{(k)}(p_1, p_2, \dots, p_n) \\ &= \left\langle \left\{ \sqrt[a]{1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k u_{i_j} \right)^q \right) \right)^{\frac{1}{C_n^k}}} \right\}^{\frac{1}{k}}, \right. \\ & \quad \left. \left\{ \sqrt[a]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (v_{i_j})^q) \right) \right)^{\frac{1}{C_n^k}}} \right)^{\frac{1}{k}}} \right\} \right\rangle. \end{aligned} \quad (11)$$

*Proof* Based on the operational laws of the Hq-ROFEs, we have

$$\bigotimes_{j=1}^k p_{i_j} = \left\langle \left\{ \prod_{j=1}^k u_{i_j} \right\}, \left\{ \sqrt[a]{1 - \prod_{j=1}^k (1 - (v_{i_j})^q)} \right\} \right\rangle$$

and

$$\begin{aligned} & \bigoplus_{1 \leq i_1 \leq \dots \leq i_k \leq n} (\otimes_{j=1}^k p_{i_j}) \\ &= \left\langle \left\{ \sqrt[a]{1 - \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k u_{i_j} \right)^q \right)} \right\}, \left\{ \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \sqrt[a]{1 - \prod_{j=1}^k (1 - (v_{i_j})^q)} \right\} \right\rangle. \end{aligned}$$

Then we obtain

$$\begin{aligned} & \frac{1}{C_n^k} \bigoplus_{1 \leq i_1 \leq \dots \leq i_k \leq n} (\otimes_{j=1}^k p_{i_j}) \\ &= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k u_{i_j} \right)^q \right) \right)^{\frac{1}{C_n^k}}} \right\}, \right. \\ & \quad \left. \left\{ \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \sqrt[q]{1 - \prod_{j=1}^k (1 - (v_{i_j})^q)} \right)^{\frac{1}{C_n^k}} \right\} \right\rangle. \end{aligned}$$

Therefore

$$\begin{aligned}
 & Hq\text{-ROFMSM}^{(k)}(p_1, p_2, \dots, p_n) \\
 &= \left\langle \left\{ \left( \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k u_{i_j} \right)^q \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}} \right\}, \right. \\
 & \quad \left. \left\{ \sqrt[a]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (v_{i_j})^q) \right) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}} \right\} \right\rangle.
 \end{aligned}$$

In what follows, we shall investigate some desirable properties of Hq-ROFMSM.

**Theorem 2** (Idempotency) Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs. If all  $p_i$  ( $i = 1, 2, \dots, n$ ) are equal, i.e.,  $p_i = p = \langle \{u\}, \{v\} \rangle$  for all  $i$ , then

$$Hq\text{-ROFMSM}^{(k)}(p_1, p_2, \dots, p_n) = \langle \{u\}, \{v\} \rangle.$$

*Proof* Since  $p = \langle \{u\}, \{v\} \rangle$ , based on Theorem 1, we have

$$\begin{aligned}
 & Hq\text{-ROFMSM}^{(k)}(p, p, \dots, p) \\
 &= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k u \right)^q \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}} \right\}, \right. \\
 & \quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (v_{i_j})^q) \right) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}} \right\} \right\rangle \\
 &= \left\langle \left\{ \left( \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} (1 - (u)^{qk}) \right)^{\frac{1}{C_n^k}}} \right)^{\frac{1}{k}} \right\}, \right. \\
 & \quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} (1 - (1 - v^q)^k) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}} \right\} \right\rangle \\
 &= \left\langle \left\{ \left( \sqrt[q]{1 - \left( (1 - (u)^{qk})^{C_n^k} \right)^{\frac{1}{C_n^k}}} \right)^{\frac{1}{k}} \right\}, \left\{ \sqrt[q]{1 - \left( 1 - \left( (1 - (1 - v^q)^k)^{C_n^k} \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}} \right\} \right\rangle \\
 &= \left\langle \left\{ \left( \sqrt[a]{1 - (1 - (u)^{qk})} \right)^{\frac{1}{k}} \right\}, \left\{ \sqrt[a]{1 - \left( 1 - (1 - (1 - v^q)^k) \right)^{\frac{1}{k}}} \right\} \right\rangle
 \end{aligned}$$

$$\begin{aligned}
&= \left\langle \left\{ \left( \sqrt[q]{(u)^{qk}} \right)^{\frac{1}{k}} \right\}, \left\{ \sqrt[q]{v^q} \right\} \right\rangle \\
&= \langle \{u\}, \{v\} \rangle,
\end{aligned}$$

which completes the proof of Theorem 2.

**Theorem 3** (Commutativity) Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, and  $p'_i = \langle g'_i, h'_i \rangle$  ( $i = 1, 2, \dots, n$ ) is any permutation of  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ), then

$$Hq\text{-ROFMSM}^{(k)}(p_1, p_2, \dots, p_n) \leq Hq\text{-ROFMSM}^{(k)}(p'_1, p'_2, \dots, p'_n)$$

*Proof* Since  $p'_i = \langle g'_i, h'_i \rangle$  ( $i = 1, 2, \dots, n$ ) is any permutation of  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ), based on the definition of Hq-ROFMSM, we have

$$\begin{aligned}
&Hq\text{-ROFMSM}^{(k)}(p_1, p_2, \dots, p_n) \\
&= \left( \frac{\bigoplus_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( \bigotimes_{j=1}^k p_{i_j} \right)}{C_n^k} \right)^{\frac{1}{k}} = \left( \frac{\bigoplus_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( \bigotimes_{j=1}^k p'_{i_j} \right)}{C_n^k} \right)^{\frac{1}{k}} \\
&= Hq\text{-ROFMSM}^{(k)}(p'_1, p'_2, \dots, p'_n).
\end{aligned}$$

Thus, the proof is complete.

**Theorem 4** (Monotonicity) Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) and  $p'_i = \langle g'_i, h'_i \rangle$  ( $i = 1, 2, \dots, n$ ) be two collections of Hq-ROFEs. If  $p_i \leq p'_i$  for all  $i$ , then

$$Hq\text{-ROFMSM}^{(k)}(p_1, p_2, \dots, p_n) \leq Hq\text{-ROFMSM}^{(k)}(p'_1, p'_2, \dots, p'_n).$$

*Proof* Since  $k \geq 1, u_i \geq u'_i \geq 0, v'_i \geq v_i \geq 0$ , we have  $u_i \geq u'_{i_j} \geq 0, v'_{i_j} \geq v_{i_j} \geq 0$ . Based on the assumption condition, for all  $i, j$  ( $i = 1, 2, \dots, n; j = 1, 2, \dots, k$ ), we obtain

$$\begin{aligned}
&\prod_{j=1}^k u_{i_j} \geq \prod_{j=1}^k u'_{i_j} \\
&\Rightarrow 1 - \left( \prod_{j=1}^k u_{i_j} \right)^q \leq 1 - \left( \prod_{j=1}^k u'_{i_j} \right)^q \\
&\Rightarrow \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k u_{i_j} \right)^q \right) \leq \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k u'_{i_j} \right)^q \right) \\
&\Rightarrow \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k u_{i_j} \right)^q \right) \right)^{\frac{1}{C_n^k}}} \\
&\leq \left( \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k u'_{i_j} \right)^q \right) \right)^{\frac{1}{C_n^k}}} \right)^{\frac{1}{k}}.
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
v'_{i_j} &\geq v_{i_j} \\
\Rightarrow 1 - (v'_{i_j})^q &\leq 1 - (v_{i_j})^q \\
\Rightarrow 1 - \prod_{j=1}^k (1 - (v'_{i_j})^q) &\geq 1 - \prod_{j=1}^k (1 - (v_{i_j})^q) \\
\Rightarrow \left( \prod_{1 \leq i \leq \dots \leq \omega_i \leq n} \left( 1 - \prod_{j=1}^k (1 - (v'_{i_j})^q) \right) \right)^{\frac{1}{c_k^k}} &\geq \left( \prod_{1 \leq i \leq \dots \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (v_{i_j})^q) \right) \right)^{\frac{1}{c_n^k}} \\
\Rightarrow \left( 1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (v'_{i_j})^q) \right) \right)^{\frac{1}{c_n^k}} \right)^{\frac{1}{k}} & \\
\leq \left( 1 - \left( \prod_{1 \leq i \leq \dots \leq i \leq n} \left( 1 - \prod_{j=1}^k (1 - (v_{i_j})^q) \right) \right)^{\frac{1}{c_n^k}} \right)^{\frac{1}{k}} & \\
\Rightarrow \sqrt[q]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq \dots \leq \omega_i \leq n} \left( 1 - \prod_{j=1}^k (1 - (v'_{i_j})^q) \right) \right)^{\frac{1}{c_n^k}} \right)^{\frac{1}{k}}} & \\
\geq \sqrt[a]{1 - \left( 1 - \left( \prod_{1 \leq i \leq \dots \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (v_{i_j})^q) \right) \right)^q \right)^{\frac{1}{c_n^k}} \right)^{\frac{1}{k}}. &
\end{aligned}$$

Thus, the proof is complete.

**Theorem 5** (Boundedness) Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, and let  $p^- = \min_i p_i, p^+ = \max_i p_i$ , then

$$p^- \leq Hq\text{-ROFMSM}^{(k)}(p_1, p_2, \dots, p_n) \leq p^+.$$

*Proof* On the basis of Theorem 2 and Theorem 4, we have

$$Hq\text{-ROFMSM}^{(k)}(p_1, p_2, \dots, p_n) \geq Hq\text{-ROFMSM}^{(k)}(p^-, p^-, \dots, p^-) = p^-,$$

$$Hq\text{-ROFMSM}^{(k)}(p_1, p_2, \dots, p_n) \leq Hq\text{-ROFMSM}^{(k)}(p^+, p^+, \dots, p^+) = p^+.$$

Thus, the proof is complete.

Now, we can discuss some special cases of the Hq-ROFMSM operator with respect to the parameter  $k$ .

**Case 1** When  $k = 1$ , the Hq-ROFMSM operator reduces to the Hesitant q-rung orthopair

fuzzy average operator as follows:

$$\begin{aligned}
 & Hq\text{-ROFMSM}^{(1)}(p_1, p_2, \dots, p_n) \\
 &= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq n} \left( 1 - \left( \prod_{j=1}^1 (u_{i_j})^{w_{i_j}} \right)^q \right) \right)^{\frac{1}{C_n^1}}} \right)^{\frac{1}{1}} \right\}, \right. \\
 &\quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq n} \left( 1 - \prod_{j=1}^1 (1 - (v_{i_j})^q)^{w_{i_j}} \right) \right)^{\frac{1}{C_n^1}} \right)^{\frac{1}{1}}} \right\} \right\rangle \\
 &= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq n} (1 - (u_{i_1})^q) \right)^{\frac{1}{n}}} \right\}, \left\{ \sqrt[q]{\left( \prod_{1 \leq i_1 \leq n} (1 - (1 - (v_{i_1})^q)) \right)^{\frac{1}{n}}} \right\} \right\rangle \\
 &= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq n} (1 - (u_{i_1})^q) \right)^{\frac{1}{n}}} \right\}, \left\{ \prod_{1 \leq i_1 \leq n} (v_{i_1})^{\frac{1}{n}} \right\} \right\rangle (\text{let } i_1 = i) \\
 &= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq n} (1 - (u_{i_1})^q) \right)^{\frac{1}{n}}} \right\}, \left\{ \prod_{i=1}^n (v_{i_1})^{\frac{1}{n}} \right\} \right\rangle.
 \end{aligned}$$

**Case 2** When  $k = 2$ , the Hq-ROFMSM operator reduces to the Hq-ROFBM operator as follows:

$$\begin{aligned}
 & Hq\text{-ROFMSM}^{(2)}(p_1, p_2, \dots, p_n) \\
 &= \left\langle \left\{ \left( \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq n} \left( 1 - \left( \prod_{j=1}^2 u_{i_j} \right)^q \right) \right)^{\frac{1}{C_n^2}}} \right)^{\frac{1}{2}} \right\}, \right. \\
 &\quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq n} \left( 1 - \prod_{j=1}^2 (1 - (v_{i_j})^q) \right) \right)^{\frac{1}{C_n^2}}} \right)^{\frac{1}{2}}} \right\} \right\rangle \\
 &= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq n} (1 - (u_{i_1} u_{i_2})^q) \right)^{\frac{2}{n(n-1)}}} \right\}, \right. \\
 &\quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq n} (1 - (1 - (v_{i_1})^q) (1 - (v_{i_2})^q)) \right)^{\frac{2}{(n-1)}} \right)^{\frac{1}{2}}} \right\} \right\rangle
 \end{aligned}$$

$$\begin{aligned}
&= \left\langle \left\{ \left( \sqrt[q]{1 - \left( \prod_{i_1, i_2=1, i_1 \neq i_2}^n (1 - (u_{i_1} u_{i_2})^q) \right)^{\frac{1}{2} \cdot \frac{2}{n(n-1)}}}} \right)^{\frac{1}{2}} \right\}, \right. \\
&\quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{i_1, i_2=1, i_1 \neq i_2} (1 - (1 - (v_{i_1})^q) (1 - (v_{i_2})^q)) \right)^{\frac{1}{2} \cdot \frac{2}{n(n-1)}}} \right)^{\frac{1}{2}} \right\} \right\rangle \\
&= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{i_1, i_2=1, i_1 \neq i_2}^n (1 - (u_{i_1} u_{i_2})^q) \right)^{\frac{1}{n(n-1)}}}} \right\}, \right. \\
&\quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{i_1, i_2=1, i_1 \neq i_2} (1 - (1 - (v_{i_1})^q) (1 - (v_{i_2})^q)) \right)^{\frac{1}{n(n-1)}}} \right)^{\frac{1}{2}} \right\} \right\rangle \\
&= Hq\text{-ROFBM}^{(1,1)}(p_1, p_2, \dots, p_n).
\end{aligned}$$

**Case 3** When  $k = n$ , the Hq-ROFMSM operator reduces to the hesitant q-rung orthopair fuzzy geometric mean operator as follows:

$$\begin{aligned}
&Hq\text{-ROFMSM}^{(n)}(p_1, p_2, \dots, p_n) \\
&= \left\langle \left\{ \left( \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq n} \left( 1 - \left( \prod_{j=1}^n u_{i_j} \right)^q \right)^{\frac{1}{c_n^n}}}} \right)^{\frac{1}{n}} \right\}, \right. \\
&\quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq n} \left( 1 - \prod_{j=1}^n (1 - (v_{i_j})^q) \right)^{\frac{1}{c_n^n}} \right)^{\frac{1}{n}} \right)^{\frac{1}{n}} \right\} \right\rangle \\
&= \left\langle \left\{ \left( \sqrt[q]{1 - \left( 1 - \left( \prod_{j=1}^n u_{i_j} \right)^q \right)^{\frac{1}{c_n^n}}} \right)^{\frac{1}{n}} \right\}, \right. \\
&\quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( 1 - \prod_{j=1}^n (1 - (v_{i_j})^q) \right)^{\frac{1}{c_n^n}} \right)^{\frac{1}{n}}} \right\} \right\rangle (\text{let } i_j = i) \\
&= \left\langle \left\{ \left( \prod_{j=1}^n u_{i_j} \right)^{\frac{1}{n}} \right\}, \left\{ \sqrt[q]{1 - \left( \prod_{j=1}^n (1 - (v_{i_j})^q) \right)^{\frac{1}{n}}} \right\} \right\rangle.
\end{aligned}$$

### 3.2 Hesitant Q-rung Orthopair Fuzzy WMSM Aggregation Operators

**Definition 16** Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, and  $w = (w_1, w_2, \dots, w_n)^T$  be the weight vector of  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ), and  $w_j > 0$ ,  $\sum_{j=1}^n w_j = 1$ , the Hq-ROFWMSM operator is a mapping Hq-ROFWMSM:

$Hq\text{-ROFE}^n \rightarrow Hq\text{-ROFE}$  can be defined as

$$Hq\text{-ROFWMSM}^{(k)}(p_1, p_2, \dots, p_n) = \left( \frac{\bigotimes_{j=1}^k (p_{i_j})^{w_{i_j}}}{C_n^k} \right)^{\frac{1}{k}}. \quad (12)$$

**Theorem 6** Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, then the aggregation result using Hq-ROFWMSM operator is also an Hq-ROFE and

$$\begin{aligned} & Hq\text{-ROFWMSM}^{(k)}(p_1, p_2, \dots, p_n) \\ &= \left\{ \int \left( \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k (u_{i_j})^{w_{i_j}} \right)^q \right) \right)^{\frac{1}{C_n^k}}} \right)^{\frac{1}{k}} \right\} \\ & \quad \left\{ \sqrt[q]{ \left( \left( 1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (v_{i_j})^q)^{w_{i_j}} \right) \right)^{\frac{1}{C_n^k}} \right) \right)^{\frac{1}{k}} \right) } \right\} \end{aligned} \quad (13)$$

*Proof* Based on the operational laws of the Hq-ROFEs, we have

$$\bigotimes_{j=1}^k (p_{i_j})^{w_{i_j}} = \left\langle \left\{ \prod_{j=1}^k (u_{i_j})^{w_{i_j}} \right\}, \left\{ q \sqrt[q]{1 - \prod_{j=1}^k (1 - (v_{i_j})^q)^{w_{i_j}}} \right\} \right\rangle$$

and

$$\begin{aligned} & \bigoplus_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( \bigotimes_{j=1}^k (p_{i_j})^{w_{i_j}} \right) \\ &= \left\langle \left\{ g \sqrt[q]{1 - \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k (u_{i_j})^{w_{i_j}} \right)^q \right)} \right\}, \right. \\ & \quad \left. \left\{ \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \sqrt[q]{1 - \prod_{j=1}^k (1 - (v_{i_j})^q)^{w_{i_j}}} \right)^{\frac{1}{C_n^k}} \right\} \right\rangle. \end{aligned}$$

Then we obtain

$$\begin{aligned} & \frac{1}{C_n^k} \bigoplus_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( \bigotimes_{j=1}^k (p_{i_j})^{w_{i_j}} \right) \\ &= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k (u_{i_j})^{w_{i_j}} \right)^q \right) \right)^{\frac{1}{C_n^k}}} \right\}, \right. \end{aligned}$$

$$\left\langle \left\{ \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \sqrt[q]{1 - \prod_{j=1}^k (1 - (v_{i_j})^q)^{w_{i_j}}} \right\} \right\rangle.$$

Therefore

$$\begin{aligned} & Hq\text{-ROFMSM}^{(k)}(p_1, p_2, \dots, p_n) \\ &= \left\langle \left\{ \left( \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k (u_{i_j})^{w_{i_j}} \right)^q \right) \right)^{\frac{1}{c_n^k}}} \right)^{\frac{1}{k}} \right\} \right. \\ & \quad \left. \left\{ \sqrt[a]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (v_{i_j})^q)^{w_{i_j}} \right) \right)^{\frac{1}{c_n^k}}} \right)^{\frac{1}{k}}} \right\} \right\rangle. \end{aligned}$$

Now, we can discuss some special cases of the Hq-ROFWMSM operator with respect to the parameter  $k$ .

**Case 1** When  $k = 1$ , the Hq-ROFWMSM operator reduces to the Hesitant q-rung orthopair fuzzy average operator as follows:

$$\begin{aligned} & Hq\text{-ROFWMSM}^{(1)}(p_1, p_2, \dots, p_n) \\ &= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq n} \left( 1 - \left( \prod_{j=1}^1 (u_{i_j})^{w_{i_j}} \right)^q \right) \right)^{\frac{1}{c_n^1}}} \right\} \right. \\ & \quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq n} \left( 1 - \prod_{j=1}^1 (1 - (v_{i_j})^q)^{w_{i_j}} \right) \right)^{\frac{1}{c_n^1}}} \right)^{\frac{1}{1}}} \right\} \right\rangle \\ &= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{1 \leq i_i \leq n} (1 - (u_{i_1})^{q w_{i_1}}) \right)^{\frac{1}{n}}} \right\} \right. \\ & \quad \left. \left\{ \left( \prod_{1 \leq i_1 \leq n} \sqrt[q]{1 - \prod_{j=1}^1 (1 - (v_{i_1})^q)^{w_{i_1}}} \right)^{\frac{1}{n}} \right\} \right\rangle \\ &= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq n} (1 - (u_{i_1})^{q w_{i_1}}) \right)^{\frac{1}{n}}} \right\} \right. \\ & \quad \left. \left\{ \left( \prod_{1 \leq i_1 \leq n} \sqrt[q]{1 - (1 - (v_{i_1})^q)^{w_{i_1}}} \right)^{\frac{1}{n}} \right\} \right\rangle \text{ (let } i_1 = i) \\ &= \left\langle \left\{ \sqrt[q]{1 - \left( \prod_{j=1}^n (1 - (u_{i_1})^{q w_{i_1}}) \right)^{\frac{1}{n}}} \right\} \right. \\ & \quad \left. \left\{ \left( \prod_{j=1}^n \sqrt[q]{1 - (1 - (v_{i_1})^q)^{w_{i_1}}} \right)^{\frac{1}{n}} \right\} \right\rangle. \end{aligned}$$

**Case 2** When  $k = 2$ , the Hq-ROFWMSM operator reduces to the Hq-ROFWBM operator as follows:

$$\begin{aligned}
 & Hq\text{-ROFWMSM}^{(2)}(p_1, p_2, \dots, p_n) \\
 &= \left\langle \left\{ \left( \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq n} \left( 1 - \left( \prod_{j=1}^2 (u_{i_j})^{n_{i_j}} \right)^q \right) \right)^{\frac{1}{c_n^2}}} \right)^{\frac{1}{2}} \right\}, \right. \\
 &\quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq n} \left( 1 - \prod_{j=1}^2 (1 - (\nu_{i_j})^q)^{n_{i_j}} \right) \right)^{\frac{1}{c_n^2}}} \right)^{\frac{1}{2}} \right\} \right\rangle \\
 &= \left\langle \left\{ \left( \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq n} \left( 1 - ((u_{i_1})^{\nu_{i_1}} (u_{i_2})^{\nu_{i_1}})^q \right) \right)^{\frac{2}{n(n-1)}}} \right)^{\frac{1}{2}} \right\}, \right. \\
 &\quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq n} (1 - (1 - (\nu_{i_1})^q)^{\nu_{i_1}} (1 - (\nu_{i_2})^q)^{w_{i_1}}) \right)^{\frac{2}{n(n-1)}}} \right)^{\frac{1}{2}} \right\} \right\rangle \\
 &= \left\langle \left\{ \left( \sqrt[q]{1 - \left( \prod_{i_1, i_2=1, i_1 \neq i_2}^n (1 - ((u_{i_1})^{\nu_{i_1}} (u_{i_2})^{\nu_{i_1}})^q) \right)^{\frac{1}{2} \cdot \frac{2}{n(n-1)}}} \right)^{\frac{1}{2}} \right\}, \right. \\
 &\quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{i_1, i_2=1, i_1 \neq i_2} (1 - (1 - (\nu_{i_1})^q)^{w_{i_1}} (1 - (\nu_{i_2})^q)^{w_{i_1}}) \right)^{\frac{1}{2} \cdot \frac{2}{n(n-1)}}} \right)^{\frac{1}{2}} \right\} \right\rangle \\
 &= \left\langle \left\{ \left( \sqrt[g]{1 - \left( \prod_{i_1, i_2=1, i_1 \neq i_2}^n (1 - ((u_{i_1})^{w_{i_1}} (u_{i_2})^{w_{i_1}})^q) \right)^{\frac{1}{n(n-1)}}} \right)^{\frac{1}{2}} \right\}, \right. \\
 &\quad \left. \left\{ \sqrt[g]{1 - \left( 1 - \left( \prod_{i_1, \lambda_2=1, i_1 \neq i_2} (1 - (1 - (\nu_{i_1})^q)^{w_{i_1}} (1 - (\nu_{i_2})^q)^{w_{i_1}}) \right)^{\frac{1}{n(n-1)}}} \right)^{\frac{1}{2}} \right\} \right\rangle \\
 &= Hq\text{-ROFWBM}^{(1,1)}(p_1, p_2, \dots, p_n).
 \end{aligned}$$

**Case 3** When  $k = n$ , the Hq-ROFWMSM operator reduces to the hesitant q-rung orthopair fuzzy geometric mean operator as follows:

$$\begin{aligned}
& Hq\text{-ROFWBM}^{(n)}(p_1, p_2, \dots, p_n) \\
&= \left\langle \left\{ \left( \sqrt[q]{1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_n \leq n} \left( 1 - \left( \prod_{j=1}^n (u_{i_j})^{w_j} \right)^q \right)^{\frac{1}{C_n^n}} \right)} \right)^{\frac{1}{n}} \right\}, \right. \\
&\quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq \dots \leq i_n \leq n} \left( 1 - \prod_{j=1}^n (1 - (v_{i_j})^q)^{w_{i_j}} \right) \right)^{\frac{1}{C_n^n}} \right)} \right)^{\frac{1}{n}} \right\} \right\rangle \\
&= \left\langle \left\{ \left( \sqrt[q]{1 - \left( 1 - \left( \prod_{j=1}^n (u_{i_j})^{w_{i_j}} \right)^q \right)^{\frac{1}{C_n^n}} \right)} \right)^{\frac{1}{n}} \right\}, \right. \\
&\quad \left. \left\{ \sqrt[q]{1 - \left( 1 - \left( 1 - \prod_{j=1}^n (1 - (v_{i_j})^q)^{w_{i_j}} \right)^{\frac{1}{C_n^n}} \right)} \right)^{\frac{1}{n}} \right\} \right\rangle \quad (\text{let } i_j = i) \\
&= \left\langle \left\{ \left( \prod_{j=1}^n (u_{i_j})^{w_{i_j}} \right)^{\frac{1}{n}} \right\}, \left\{ \sqrt[q]{1 - \left( \prod_{j=1}^n (1 - (v_{i_j})^q)^{w_{i_j}} \right)^{\frac{1}{C_n^n}} \right)} \right)^{\frac{1}{n}} \right\} \right\rangle.
\end{aligned}$$

### 3.3 Hesitant Q-rung Orthopair Fuzzy DMSM Aggregation Operators

In the following, we shall develop the Hq-ROFDMSM operator based on the operations of Hq-ROFEs.

**Definition 17** Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs. The Hq-ROFDMSM operator is a mapping Hq-ROFDMSM:

$Hq\text{-ROFE}^n \rightarrow Hq\text{-ROFE}$  can be defined as

$$Hq\text{-ROFDMSM}^{(k)}(p_1, p_2, \dots, p_n) = \frac{1}{k} \left( \bigotimes_{1 \leq i_1 \leq \dots \leq i_k \leq n} (\oplus_{j=1}^k p_{i_j})^{\frac{1}{C_n^k}} \right). \quad (14)$$

**Theorem 7** Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, then the aggregation result using Hq-ROFDMSM operator is also an Hq-ROFE and

$$\begin{aligned}
& Hq\text{-ROFDMSM}^{(k)} M^{(k)}(p_1, p_2, \dots, p_n) \\
&= \left\langle \left\{ \sqrt[a]{1 - \left( 1 - \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (u_{i_j})^q)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}} \right)} \right\}, \right. \\
&\quad \left. \left\{ \left( \sqrt[a]{1 - \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k v_{i_j} \right)^q \right)^{\frac{1}{C_n^k}} \right)} \right)^{\frac{1}{k}} \right\} \right\rangle. \quad (15)
\end{aligned}$$

*Proof* Based on the operational laws of the Hq-ROFEs, we have

$$\oplus_{j=1}^k p_{i_j} = \left\langle \left\{ \sqrt[k]{1 - \prod_{j=1}^k (1 - (u_{i_j})^q)} \right\}, \left\{ \prod_{j=1}^k v_{i_j} \right\} \right\rangle$$

and

$$\left( \bigoplus_{j=1}^k p_{i_j} \right)^{\frac{1}{c_n^k}} = \left\langle \left\{ \left( \sqrt[q]{1 - \prod_{j=1}^k (1 - (u_{i_j})^q)} \right)^{\frac{1}{c_n^k}} \right\}, \left\{ \sqrt[q]{1 - \left( 1 - \left( \prod_{j=1}^k v_{i_j} \right)^q \right)^{\frac{1}{c_n^k}}} \right\} \right\rangle.$$

Then we obtain

$$\begin{aligned} & \bigotimes_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( \bigoplus_{j=1}^k p_{i_j} \right)^{\frac{1}{c_n^k}} \\ &= \left\langle \left\{ \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( \sqrt[q]{1 - \prod_{j=1}^k (1 - (u_{i_j})^q)} \right)^{\frac{1}{c_n^k}} \right\}, \right. \\ & \quad \left. \left\{ \sqrt[q]{1 - \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k v_{i_j} \right)^q \right)^{\frac{1}{c_n^k}}} \right\} \right\rangle. \end{aligned}$$

Therefore

$$\begin{aligned} & Hq\text{-ROFDMSM}^{(k)}(p_1, p_2, \dots, p_n) \\ &= \left\langle \left\{ \sqrt[a]{1 - \left( 1 - \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (u_{i_j})^q) \right)^{\frac{1}{c_n^k}} \right)^{\frac{1}{k}}} \right\}, \right. \\ & \quad \left. \left\{ \left( \sqrt[q]{1 - \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k v_{i_j} \right)^q \right)^{\frac{1}{c_n^k}}} \right)^{\frac{1}{k}} \right\} \right\rangle. \end{aligned}$$

In what follows, we shall investigate some desirable properties of Hq-ROFDMSM.

**Theorem 8** (Idempotency) Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, If all  $p_i$  ( $i = 1, 2, \dots, n$ ) are equal, i.e.,  $p_i = p = \langle \{u\}, \{v\} \rangle$  for all  $i$ , then

$$Hq\text{-ROFDMSM}^{(k)}(p_1, p_2, \dots, p_n) = \langle \{u\}, \{v\} \rangle.$$

**Theorem 9** (Commutativity) Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, and  $p'_i = \langle g'_i, h'_i \rangle$  ( $i = 1, 2, \dots, n$ ) is any permutation of  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ), then

$$Hq\text{-ROFDMSM}^{(k)}(p_1, p_2, \dots, p_n) \leq Hq\text{-ROFDMSM}^{(k)}(p'_1, p'_2, \dots, p'_n).$$

**Theorem 10** (Monotonicity) Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) and  $p'_i = \langle g'_i, h'_i \rangle$  ( $i = 1, 2, \dots, n$ ) be two collections of Hq-ROFEs. If  $p_i \leq p'_i$  for all  $i$ , then

$$Hq\text{-ROFDMSM}^{(k)}(p_1, p_2, \dots, p_n) \leq Hq\text{-ROFDMSM}^{(k)}(p'_1, p'_2, \dots, p'_n).$$

**Theorem 11** (Boundedness) Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, and let  $p^- = \min_i p_i, p^+ = \max_i p_i$ , then

$$p^- \leq Hq\text{-ROFDMSM}^{(k)}(p_1, p_2, \dots, p_n) \leq p^+.$$

### 3.4 Hesitant Q-rung Orthopair Fuzzy WDMSM Aggregation Operators

In the above section, it can be seen that the Hq-ROFDMSM operator does not consider the importance of the aggregated arguments. However, in many real practical situations, especially in MADM, the weights of attributes play an important role in the process of aggregation. To overcome the limitation of the Hq-ROFDMSM operator, we shall propose the q-ROFDMSM operator as follows.

**Definition 18** Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, and  $w = (w_1, w_2, \dots, w_n)^T$  be the weight vector of  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ), and  $w_j > 0$ ,  $\sum_{j=1}^n w_j = 1$ , the Hq-ROFWMSM operator is a mapping Hq-ROFWMSM:

$Hq\text{-ROFE}^n \rightarrow Hq\text{-ROFE}$  can be defined as

$$Hq\text{-ROFWDMSM}^{(k)}(p_1, p_2, \dots, p_n) = \frac{1}{k} \left( \bigotimes_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( \bigoplus_{j=1}^k w_{i_j} p_{i_j} \right)^{\frac{1}{C_n^k}} \right). \quad (16)$$

**Theorem 12** Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, then the aggregation result using Hq-ROFWDMSM operator is also a Hq-ROFE and

$$\begin{aligned} & Hq\text{-ROFWDMSM}^{(k)}(p_1, p_2, \dots, p_n) \\ &= \left\langle \left\{ \sqrt[k]{1 - \left( 1 - \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (u_{i_j})^q)^{w_{i_j}} \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}} \right\}, \right. \\ & \quad \left. \left\{ \left( \sqrt[k]{1 - \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k (v_{i_j})^{w_{i_j}} \right)^q \right)^{\frac{1}{C_n^k}}} \right)^{\frac{1}{k}} \right\} \right\rangle. \end{aligned} \quad (17)$$

*Proof* Based on the operational laws of the Hq-ROFEs, we have

$$\bigoplus_{j=1}^k (w_{i_j} p_{i_j}) = \left\langle \left\{ \sqrt[k]{1 - \prod_{j=1}^k (1 - (u_{i_j})^q)^{w_{i_j}}} \right\}, \left\{ \prod_{j=1}^k (v_{i_j})^{w_{i_j}} \right\} \right\rangle$$

and

$$\begin{aligned} & \left( \bigoplus_{j=1}^k w_{i_j} p_{i_j} \right)^{\frac{1}{C_n^k}} \\ &= \left\langle \left\{ \sqrt[k]{1 - \prod_{j=1}^k (1 - (u_{i_j})^q)^{w_{i_j}}} \right\}^{\frac{1}{C_n^k}}, \left\{ \sqrt[k]{1 - \left( 1 - \left( \prod_{j=1}^k (v_{i_j})^{w_{i_j}} \right)^q \right)^{\frac{1}{C_n^k}}} \right\} \right\rangle, \end{aligned}$$

Then we obtain

$$\begin{aligned} & \bigotimes_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( \bigoplus_{j=1}^k w_{i_j} p_{i_j} \right)^{\frac{1}{C_n^k}} \\ &= \left\langle \left\{ \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( \sqrt[q]{1 - \prod_{j=1}^k (1 - (u_{i_j})^q)^{w_{i_j}}} \right)^{\frac{1}{C_n^k}} \right\}, \right. \\ & \quad \left. \left\{ \sqrt[q]{1 - \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k (v_{i_j})^{w_{i_j}} \right)^q \right)^{\frac{1}{C_n^k}}} \right\} \right\rangle. \end{aligned}$$

Therefore

$$\begin{aligned} & Hq\text{-ROFWMSM}^{(k)}(p_1, p_2, \dots, p_n) \\ &= \left\langle \left\{ \sqrt[q]{1 - \left( 1 - \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (u_{i_j})^q)^{w_{i_j}} \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}}} \right\}, \right. \\ & \quad \left. \left\{ \left( \sqrt[q]{1 - \prod_{1 \leq i_1 \leq \dots \leq i_k \leq n} \left( 1 - \left( \prod_{j=1}^k (v_{i_j})^{w_{i_j}} \right)^q \right)^{\frac{1}{C_n^k}}} \right)^{\frac{1}{k}} \right\} \right\rangle. \end{aligned}$$

#### 4 Models for MADM with Hesitant Q-rung Orthopair Fuzzy Information

In this section, we shall utilize the hesitant q-rung orthopair fuzzy maclaurin symmetric mean aggregation operators to multiple attribute decision making with Hq-ROFEs information.

For a MADM problem, suppose  $A = \{A_1, A_2, \dots, A_m\}$  ( $i = 1, 2, \dots, m$ ) is the set of alternatives, let  $C = \{C_1, C_2, \dots, C_n\}$  be the set of attributes. Let  $w = (w_1, w_2, \dots, w_n)$  be the weight vector of attributes, where  $w_j \geq 0, j = 1, 2, \dots, n, \sum_{i=1}^n w_j = 1$ . Suppose that  $P = (p_{ij})_{m \times n} = \langle \{u_{ij}\}, \{v_{ij}\} \rangle_{m \times n}$  ( $i = 1, 2, \dots, m, j = 1, 2, \dots, n$ ) is the hesitant q-rung orthopair fuzzy decision matrix, where  $u_{ij}$  indicates the degree that the alternative  $A_i$  satisfies the attribute  $C_j$  given by the decision maker,  $v_{ij}$  indicates the degree that the alternative  $A_i$  doesn't satisfies the attribute  $C_j$  given by the decision maker,  $u_{ij} \in [0, 1], v_{ij} \in [0, 1], (u_{ij})^q + (v_{ij})^q \leq 1, i = 1, 2, \dots, m, j = 1, 2, \dots, n$ .

In the following, we apply the Hq-ROFWMSM operator to the multiple attribute decision making with Hq-ROFEs information.

**Step 1** We utilize the decision information given in matrix  $P$ , and the Hq-ROFWMSM operator

$$\begin{aligned}
 & Hq\text{-ROFWMSM } M^{(k)}(p_1, p_2, \dots, p_n) \\
 &= \left\langle \sqrt[3]{1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq 4} \left( 1 - \left( \prod_{j=1}^2 (u_{i_j})^{w_{i_j}} \right)^3 \right)^{\frac{1}{C_4^2}} \right)^{\frac{1}{2}}}, \right. \\
 & \quad \left. \sqrt[q]{\left\{ 1 - \left( 1 - \left( \prod_{1 \leq i \leq \dots \leq i_k \leq n} \left( 1 - \prod_{j=1}^k (1 - (v_{i_j})^q)^{w_{i_j}} \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}} \right\}} \right\rangle
 \end{aligned}$$

to derive the overall preference values  $p_i$  ( $i = 1, 2, \dots, m$ ) of the alternative  $A_i$ .

**Step 2** Calculate the scores  $S(p_i)$  ( $i = 1, 2, \dots, m$ ) of the overall Hq-ROFEs  $p_i$  ( $i = 1, 2, \dots, m$ ) to rank all the alternatives  $A_i$  ( $i = 1, 2, \dots, m$ ) and then to select the best one(s).

**Step 3** Rank all the alternatives  $A_i$  ( $i = 1, 2, \dots, m$ ) and select the best one(s) in accordance with  $S(p_i)$  ( $i = 1, 2, \dots, m$ ).

**Step 4** End.

## 5 Numerical Example and Comparative Analysis

### 5.1 Numerical Example

In this section, a numerical example is presented to show potential evaluation of quality of digital government services with hesitant q-rung orthopair fuzzy information in order to illustrate the method proposed in this paper.

In recent years, with “Internet plus government service” as the starting point, the “run at most once” reform as the overall traction, and the integrated data platform as the key support, Chongqing has achieved remarkable results in government digital governance, which is very representative. In addition, Chongqing has achieved significant results in economic development and political reform due to its geographical advantage, and has a certain exemplary role in the country. Therefore, this study takes Chongqing as an example to comprehensively evaluate the quality of its digital government services.

There are five Chongqing government affairs WeChat official accounts (Chongqing Medical, Chongqing Transportation, Chongqing Weather, Chongqing Public Security Bureau, Chongqing Taxation)  $A_i$  ( $i = 1, 2, 3, 4, 5$ ) to select. The expert selects four attribute to evaluate the five possible official accounts:  $C_1$  is platform design quality;  $C_2$  is the information and administrative services;  $C_3$  the information technology;  $C_4$  is the government WeChat information environment. The weighting vector is  $w = (0.2, 0.1, 0.3, 0.4)^T$ . The five official accounts  $A_i$  ( $i = 1, 2, 3, 4, 5$ ) are to be evaluated using the hesitant q-rung orthopair fuzzy information by the decision maker under the above four attributes, and the hesitant q-rung orthopair fuzzy decision matrix  $P = (p_{ij})_{5 \times 4} = \langle \{u_{ij}\}, \{v_{ij}\} \rangle_{5 \times 4}$  is shown in Table 1.

**Table 1** Hesitant q-rung orthopair fuzzy set

|       | $C_1$                                        | $C_2$                                        | $C_3$                                        | $C_4$                                        |
|-------|----------------------------------------------|----------------------------------------------|----------------------------------------------|----------------------------------------------|
| $A_1$ | $\langle \{0.7, 0.8\}, \{0.5, 0.6\} \rangle$ | $\langle \{0.3, 0.4\}, \{0.8, 0.9\} \rangle$ | $\langle \{0.7\}, \{0.5, 0.6\} \rangle$      | $\langle \{0.8, 0.9\}, \{0.3, 0.4\} \rangle$ |
| $A_2$ | $\langle \{0.5, 0.6\}, \{0.7\} \rangle$      | $\langle \{0.4, 0.5\}, \{0.7, 0.8\} \rangle$ | $\langle \{0.80.0.9\}, \{0.3, 0.4\} \rangle$ | $\langle \{0.7, 0.8\}, \{0.5, 0.6\} \rangle$ |
| $A_3$ | $\langle \{0.5, 0.6\}, \{0.6, 0.7\} \rangle$ | $\langle \{0.7, 0.8\}, \{0.3, 0.4\} \rangle$ | $\langle \{0.6, 0.7\}, \{0.5, 0.6\} \rangle$ | $\langle \{0.3, 0.4\}, \{0.7, 0.9\} \rangle$ |
| $A_4$ | $\langle \{0.8\}, \{0.3, 0.4\} \rangle$      | $\langle \{0.6, 0.7\}, \{0.7\} \rangle$      | $\langle \{0.4, 0.5\}, \{0.7, 0.8\} \rangle$ | $\langle \{0.5, 0.6\}, \{0.7\} \rangle$      |
| $A_5$ | $\langle \{0.3, 0.4\}, \{0.8, 0.9\} \rangle$ | $\langle \{0.7, 0.8\}, \{0.5, 0.6\} \rangle$ | $\langle \{0.4, 0.5\}, \{0.6, 0.7\} \rangle$ | $\langle \{0.6, 0.7\}, \{0.5\} \rangle$      |

In order to select the most desirable account, we utilize the Hq-ROFWMSM operator to develop an approach to multiple attribute decision making problems with hesitant q-rung orthopair fuzzy information, which can be described as following.

**Step 1** We utilize the Hq-ROFWMSM operator to aggregate all the hesitant q-rung orthopair fuzzy information  $p_{ij}$  ( $i = 1, 2, 3, 4, 5; j = 1, 2, 3, 4$ ) into the overall hesitant q-rung orthopair fuzzy values  $p_i$  ( $i = 1, 2, 3, 4$ ). Taking the  $A_1$  as the example, here let  $k = 2, q = 3$ , we have

$$\begin{aligned}
 p_1 &= Hq-ROFWMSM^{(2)}(p_{11}, p_{12}, p_{13}, p_{14}, p_{15}) \\
 &= \left\langle \left\{ \left( \sqrt[3]{1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq 4} \left( 1 - \left( \prod_{j=1}^2 (u_{i_j})^{w_{i_j}} \right)^3 \right) \right)^{\frac{1}{c_4^2}} \right)^{\frac{1}{2}} \right\}, \right. \\
 &\quad \left. \left\{ \sqrt[3]{1 - \left( 1 - \left( \prod_{1 \leq i_1 \leq i_2 \leq 4} \left( 1 - \prod_{j=1}^2 \left( 1 - (v_{1_j})^3 \right)^{w_1} \right) \right)^{\frac{1}{c_4^2}} \right)^{\frac{1}{2}} \right\} \right\rangle \\
 &= \langle \{0.8285, 0.8508, 0.8397, 0.8616, 0.8415, 0.8641, 0.8525, 0.8745\}, \\
 &\quad 0.3786, 0.3963, 0.4047, 0.4211, 0.4104, 0.4278, 0.4365, 0.4524, 0.4 \\
 &\quad 0.4158, 0.4253, 0.4396, 0.4317, 0.4469, 0.4568, 0.4702\} \rangle.
 \end{aligned}$$

**Step 2** Calculate the score values  $S(p_i)$  ( $i = 1, 2, 3, 4, 5$ ) of the overall hesitant q-rung orthopair fuzzy preference values  $p_i$  ( $i = 1, 2, 3, 4, 5$ ).

$$s(p_1) = 0.5405, s(p_2) = 0.5085, s(p_3) = 0.3102, s(p_4) = 0.3584, s(p_5) = 0.2785.$$

**Step 3** Rank all the account  $A_i$  ( $i = 1, 2, 3, 4, 5$ ) in accordance with the score value  $s(p_i)$  of the overall fuzzy preference value  $p_i$ :  $A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$ . And thus, the most desirable account is  $A_1$ .

## 5.2 The Effect of the Parameters $k$ on Decision-making Result

In order to reflect the influence with the different value of the parameter  $k$ , we use a different parameter  $k$  in our proposed method to rank the alternatives. The ranking results are shown in Table 2 (let  $q = 3$ ).

**Table 2** Ordering of the alternatives using different parameter  $k$  in Hq-ROFWMSM operator

| Parameter value | $A_1$  | $A_2$  | $A_3$  | $A_4$  | $A_5$  | Ranking order                                 |
|-----------------|--------|--------|--------|--------|--------|-----------------------------------------------|
| $k = 1$         | 0.5616 | 0.5202 | 0.3811 | 0.3782 | 0.2812 | $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ |
| $k = 2$         | 0.5405 | 0.5085 | 0.3102 | 0.3584 | 0.2785 | $A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$ |
| $k = 3$         | 0.5375 | 0.4988 | 0.3043 | 0.3465 | 0.2767 | $A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$ |
| $k = 4$         | 0.5144 | 0.4725 | 0.2983 | 0.2931 | 0.2623 | $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ |
| $k = 5$         | 0.4989 | 0.4558 | 0.2844 | 0.2702 | 0.2525 | $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ |

By Table 2, it is evident that the ranking order results in this example slightly differ with the change in parameters based on the decision maker's subjective preferences. This observation signifies that the Hq-ROFWMSM operator is capable of reflecting the decision maker's risk preferences. Furthermore, upon further analysis, it is observed that the score values obtained by the Hq-ROFWMSM operator decrease as the parameter increases for the same alternative.

In practical decision-making situations, decision makers have the flexibility to select the appropriate value in accordance with their risk preferences. Typically, we consider  $k = \lceil n/2 \rceil$  for computation in practical problems, where the symbol  $\lceil \cdot \rceil$  represents the round function and  $n$  is the number of attributes. This approach is not only intuitive and simple but also accounts for the decision maker's neutral risk preferences and fully considers the interrelationships among the individual arguments in this case.

### 5.3 The Effect of the Parameters $q$ on the Decision-making Result

In the following, we will discuss the influence of the parameter  $q$  on the decision-making result based on the Hq-ROFWMSM operator, and the ranking results with the different values  $q$  are shown in Table 3 (when  $k = 3$ ). From Table 3, we know that the ranking results may be different for the different parameters  $q$  by the Hq-ROFWMSM operator. However, the best alternative is  $A_1$ . The decision makers can choose the appropriate parameter value  $q$  by their preferences. Furthermore, from Table 3, we can easily know that the score functions based on the Hq-ROFWMSM operator became smaller and smaller.

**Table 3** Score values obtained by the Hq-ROFWMSM and the rankings of alternatives

| Parameter value | $A_1$  | $A_2$  | $A_3$  | $A_4$  | $A_5$  | Ranking order                                 |
|-----------------|--------|--------|--------|--------|--------|-----------------------------------------------|
| $q = 1$         | 0.5709 | 0.5344 | 0.3657 | 0.3523 | 0.2935 | $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ |
| $q = 2$         | 0.5524 | 0.5285 | 0.3289 | 0.3457 | 0.2828 | $A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$ |
| $q = 3$         | 0.5231 | 0.5137 | 0.3143 | 0.3365 | 0.2767 | $A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$ |
| $q = 4$         | 0.5136 | 0.4832 | 0.3098 | 0.3085 | 0.2558 | $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ |
| $q = 5$         | 0.5208 | 0.4768 | 0.2914 | 0.2926 | 0.2411 | $A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$ |

### 5.4 Comparative Analysis

To elaborate the effectiveness and practicability of the proposed approach, we give a comparative analysis of the developed operators with several previous decision methods, including

the hesitant q-rung orthopair fuzzy weighted averaging (Hq-ROFWA) operator and hesitant q-rung orthopair fuzzy weighted geometric (Hq-ROFWG) operator proposed by Hussain, et al.<sup>[44]</sup>. We will discuss this example in the following parts.

**Definition 19**<sup>[44]</sup> Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, and  $w = (w_1, w_2, \dots, w_n)^T$  be the weight vector of  $p_i$  ( $i = 1, 2, \dots, n$ ) with  $w_i \geq 0$  ( $i = 1, 2, \dots, n$ ) where  $w_i \in [0, 1]$  and  $\sum_{i=1}^n w_i = 1$ . The hesitant q-rung orthopair fuzzy weighted averaging (Hq-ROFWA) operator is a mapping Hq-ROFWA:

$$Hq\text{-ROFWA}(p_1, p_2, \dots, p_n) = \oplus_{i=1}^n w_i p_i = \left\{ \left\{ \sqrt[q]{1 - \prod_{i=1}^n (1 - u_i^q)^{w_i}} \right\}, \left\{ \prod_{i=1}^n v_i^{w_i} \right\} \right\}. \quad (18)$$

**Definition 20**<sup>[44]</sup> Let  $p_i = \langle g_i, h_i \rangle$  ( $i = 1, 2, \dots, n$ ) be a collection of Hq-ROFEs, and  $w = (w_1, w_2, \dots, w_n)^T$  be the weight vector of  $p_i$  ( $i = 1, 2, \dots, n$ ) with  $w_i \geq 0$  ( $i = 1, 2, \dots, n$ ) where  $w_i \in [0, 1]$  and  $\sum_{i=1}^n w_i = 1$ . The hesitant q-rung orthopair fuzzy weighted geometric (Hq-ROFWG) operator is a mapping Hq-ROFWG:

$$Hq\text{-ROFWG}(p_1, p_2, \dots, p_n) = \otimes_{i=1}^n (p_i)^{w_i} = \left\{ \left\{ \prod_{i=1}^n u_i^{w_i} \right\}, \left\{ \sqrt[q]{1 - \prod_{i=1}^n (1 - v_i^q)^{w_i}} \right\} \right\}. \quad (19)$$

By using the decision information given in this example, and the Hq-ROFWA and Hq-ROFWG operators, the aggregating results are shown in Table 4 and Table 5.

**Table 4** Score and ranking of the alternatives with Hq-ROFWA

| $x$      | score of $S(p_i)$ ( $i = 1, 2, 3, 4, 5$ )                                                   | Ranking                                       |
|----------|---------------------------------------------------------------------------------------------|-----------------------------------------------|
| $q = 1$  | $s(p_1) = 0.6423, s(p_2) = 0.6314, s(p_3) = 0.5852,$<br>$s(p_4) = 0.5968, s(p_5) = 0.5434.$ | $A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$ |
| $q = 2$  | $s(p_1) = 0.6268, s(p_2) = 0.6114, s(p_3) = 0.5232,$<br>$s(p_4) = 0.5767, s(p_5) = 0.5652.$ | $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$ |
| $q = 3$  | $s(p_1) = 0.5990, s(p_2) = 0.4577, s(p_3) = 0.5044,$<br>$s(p_4) = 0.5386, s(p_5) = 0.5847.$ | $A_1 \succ A_5 \succ A_4 \succ A_3 \succ A_2$ |
| $q = 4$  | $s(p_1) = 0.6104, s(p_2) = 0.4163, s(p_3) = 0.3732,$<br>$s(p_4) = 0.4465, s(p_5) = 0.5942.$ | $A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3$ |
| $q = 5$  | $s(p_1) = 0.6058, s(p_2) = 0.4036, s(p_3) = 0.3629,$<br>$s(p_4) = 0.4379, s(p_5) = 0.5859.$ | $A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3$ |
| $q = 6$  | $s(p_1) = 0.5924, s(p_2) = 0.5767, s(p_3) = 0.3548,$<br>$s(p_4) = 0.4238, s(p_5) = 0.3832.$ | $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$ |
| $q = 7$  | $s(p_1) = 0.5847, s(p_2) = 0.5624, s(p_3) = 0.3475,$<br>$s(p_4) = 0.4144, s(p_5) = 0.3859.$ | $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$ |
|          | $s(p_1) = 0.5794, s(p_2) = 0.5585, s(p_3) = 0.3372,$                                        | $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$ |
| $q = 8$  | $s(p_4) = 0.4001, s(p_5) = 0.3774.$<br>$s(p_1) = 0.5658, s(p_2) = 0.5469, s(p_3) = 0.3256,$ | $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$ |
| $q = 9$  | $s(p_4) = 0.3991, s(p_5) = 0.3663.$<br>$s(p_1) = 0.5416, s(p_2) = 0.5207, s(p_3) = 0.3013,$ | $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$ |
| $q = 10$ | $s(p_4) = 0.3790, s(p_5) = 0.3483.$                                                         |                                               |

**Table 5** Score and ranking of the alternatives with Hq-ROFWG

| $c$      | score of $s(p_i)$ ( $i = 1, 2, 3, 4, 5$ )                                                   | Ranking                                       |
|----------|---------------------------------------------------------------------------------------------|-----------------------------------------------|
| $q = 1$  | $s(p_1) = 0.8528, s(p_2) = 0.8312, s(p_3) = 0.7999$<br>$s(p_4) = 0.7011, s(p_5) = 0.7139.$  | $A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$ |
| $q = 2$  | $s(p_1) = 0.8134, s(p_2) = 0.7978, s(p_3) = 0.7725,$<br>$s(p_4) = 0.6909, s(p_5) = 0.6836.$ | $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ |
| $q = 3$  | $s(p_1) = 0.7985, s(p_2) = 0.7595, s(p_3) = 0.7723,$<br>$s(p_4) = 0.6724, s(p_5) = 0.6678.$ | $A_1 \succ A_3 \succ A_2 \succ A_4 \succ A_5$ |
| $q = 4$  | $s(p_1) = 0.7889, s(p_2) = 0.6574, s(p_3) = 0.7423,$<br>$s(p_4) = 0.6645, s(p_5) = 0.7652.$ | $A_1 \succ A_5 \succ A_3 \succ A_4 \succ A_2$ |
| $q = 5$  | $s(p_1) = 0.7747, s(p_2) = 0.7578, s(p_3) = 0.7345,$<br>$s(p_4) = 0.6511, s(p_5) = 0.6434$  | $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ |
| $q = 6$  | $s(p_1) = 0.7695, s(p_2) = 0.7484, s(p_3) = 0.7268,$<br>$s(p_4) = 0.6457, s(p_5) = 0.6343.$ | $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ |
| $q = 7$  | $s(p_1) = 0.7597, s(p_2) = 0.7355, s(p_3) = 0.7156,$<br>$s(p_4) = 0.6343, s(p_5) = 0.6212.$ | $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ |
| $q = 8$  | $s(p_1) = 0.7426, s(p_2) = 0.7232, s(p_3) = 0.7049,$<br>$s(p_4) = 0.6297, s(p_5) = 0.6158.$ | $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ |
| $q = 9$  | $s(p_1) = 0.7323, s(p_2) = 0.7173, s(p_3) = 0.6979,$<br>$s(p_4) = 0.6192, s(p_5) = 0.6085.$ | $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ |
| $q = 10$ | $s(p_1) = 0.7205, s(p_2) = 0.7032, s(p_3) = 0.6891,$<br>$s(p_4) = 0.6019, s(p_5) = 0.5923.$ | $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$ |

Table 4 and Table 5 reveals that regardless of the parameter values used, the same ranking results consistently identify A1 as the best account. This finding validates the reasonableness and effectiveness of the method proposed in the paper.

In comparison to the methods introduced by Hussain, et al.<sup>[44]</sup>, our proposed Hq-ROFWMSM operator possesses a desirable property of monotonically decreasing with respect to the parameter  $k$ . This allows decision-makers to select an appropriate value based on their risk preferences. If a decision-maker favors risk, a larger parameter value can be chosen, while a smaller value can be selected if risk aversion is preferred. In contrast, the models proposed by Hussain, et al.<sup>[44]</sup> lack this parameter monotonicity property, making it challenging to determine suitable parameters that reflect decision-makers' attitudes in real-world decision-making scenarios.

Another advantage of our method over the approaches presented by Hussain, et al.<sup>[44]</sup> is its ability to consider the interrelationship among multi-input arguments. In contrast, the methods proposed by Hussain, et al.<sup>[44]</sup> do not account for the interrelationships among these arguments. Therefore, our method is more versatile and general. Additionally, the methods proposed by Hussain, et al.<sup>[44]</sup> lack the capability to dynamically adjust the parameter according to decision-makers' risk attitudes, making it difficult to handle risk in practical MADM situations. Our method overcomes these limitations by utilizing the Hq-ROFWMSM operator, which provides more flexible and robust information fusion. Consequently, our method is better suited to address MADM problems where the attributes are independent. Based on these comparisons

and analyses, the Hq-ROFWMSM operator proposed in this paper outperforms the methods introduced by Hussain, et al.<sup>[44]</sup> in aggregating the Hq-ROFEs and is thus more suitable for solving MADM with Hq-ROFEs.

## 6 Conclusion

The prominent characteristic of the MSM is that it can capture the interrelationship among the multi-input arguments and can provide more flexible and robust information fusion, and based on the advantages of hesitant q-rung orthopair fuzzy sets (Hq-ROFSs), an attempt has been made to present Maclaurin symmetric mean aggregation operator under Hq-ROFS environment. The contributions of this paper are outlined below:

1) We propose some new aggregation operators for Hq-ROFS, such as the hesitant q-rung orthopair fuzzy maclaurin symmetric mean (Hq-ROFMSM) operator, the hesitant q-rung orthopair fuzzy weighted maclaurin symmetric mean (Hq-ROFWMSM) operator, the hesitant q-rung orthopair fuzzy dual maclaurin symmetric mean (Hq-ROFDMSM) operator, the hesitant q-rung orthopair fuzzy weighted dual maclaurin symmetric mean (Hq-ROFWDMSM) operator.

2) The Hq-ROFWMSM operator is utilized to develop an approach to solve MADM problems under Hq-ROFSs, and elaborated on the detailed solution steps.

3) A practical example about evaluation of WeChat official account is given to verify the practicality and validity of the proposed method by comparing different aggregation operators and parameter values.

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