

A Holistic Warfare Model: The Quantum Computation and Category Method

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Abstract Quantum computation and artificial intelligence for military applications bring new capabilities, leading to quantum warfare. In this paper, we present a Holistic Warfare Model, introduced as a high-dimensional entangled warfare category including asymmetric conflicts. Its underlying metaphysics is entangled states fusion: This is the macroscopic entanglement concept inspired by high-dimensional quantum computation with the entangled wave-functions. From this entangled view, wars and battles are seen essentially as a holistic phenomenon: The distinct operation behaviors within different battlefields tend to the worldwide war. The mathematical Holistic Warfare framework developed in this paper expresses this fundamental view of arbitrary many interaction conflicts, each of them defined by its own battle-manifold and evolution simultaneously on the planet; we call this entangled category $\mathcal{WAR}$. At last, we introduce the research directions in terms of the future wars, such as multilateral conflict, cognitive warfare, biological warfare and unmanned operation so on.

Keywords holistic warfare model; lanchester equation; artificial intelligence; quantum computation; category theory

1 Introduction

Military operations research and warfare modeling apply mathematical tools to address a variety of warfare conflicts and combat situations^[1,2]. Until recently, the great majority of works on this topic have focused on the dynamics of isolated battles or missions^[3]. However, the method cannot represent the complexity of modern warfare, and especially the nonlinearity and integrality of interactions among different involved sides and related levels within global warfare, thus impelling us to research wars not just as series of campaigns, campaigns as not just series of battles, and battles as not just as series of missions, in spatio-temporal events. Inversely, we try here to model war as any collections of overlapping battles and battles as

arbitrary collections of fusing missions, which can unfold concurrently and can interact with one another. Thus, we present the basic concept of Holistic Warfare in this paper, which attempts to realize a system method to modeling war, campaign, battle, and mission so on.

The growing complexity and interdependence (see [4]) signifies that to more reasonably design appropriate future force structure, fighting capability and operating methods, strategies and tactics, a more global approach to modeling is necessary. As a result of this need for more global methods, we seek to capture all the interdependencies between various overlapping battles and simultaneous campaigns in different theaters, and among events during an arbitrary given war, battle or mission. The intent in this paper is to construct a formal mathematical architecture through the utility of manifold, the concept of entanglement, and the framework of category to form this holistic view of war and mission, which makes up thus a national or multinational viewpoint.

It has been long affirmed that warfare emerge a deeply complex phenomenon that doesn't contain only elements of order but also elements of chaos, usually with both co-occurring^[5]. It can be recognized that, if all other things being equal, the winner will be the party which makes the least mistakes; or, to put it another way, the party which is able to manage the chaos the most effectively. We realize that complex systems lead to emergent phenomena which can only be explained through modeling and simulations if the model or simulation has captured adequately the large-scale interrelation that connect all the operations and events that collectively constitute a global framework on war and battle from a national perspective.

The central theme of this presented paper, current reflecting our holistic view of warfare, is the concept of a large-scale and complex operational fusion, consisting of many interactional battlefields, that is, an entangled states fusion of operation manifolds. In other words, we see a modern warfare as a compound system of systems, and an indivisible fusion, which comprises a series of non-local yet interacting battlefields, rather than a reducible, decoupled mechanistic components.

The entangled states fusion is a macroscopic concept inspired by high-dimensional quantum computation (see [6,7]), in which any number of entangled wave-functions can be highly-correlated and indivisible. Quantum computation and artificial intelligence^[8] for military applications bring new capabilities, leading to *quantum warfare* which uses quantum technologies for military applications that affect intelligence^[9,10], security and defense capabilities of all warfare domains and brings new military strategies, doctrines, scenarios and peace or ethics issues^[11]. The entanglement is visible in any macroscopic system or process which cannot be completely reduced to its components, but that manifests behavior only as a whole fused system^[12]. Biological systems like various swarming behaviors of and colony, bird flock and sheep herd show this emergency characteristic. By analogy, in this paper we claim that the concept of a global warfare is essentially an entangled states fusion of more than two parties at least some of whom are in a state of conflict; the war cannot exist without conflicting parties, even if not all parties need to concurrently be in conflict at any one time. Any attempt to reduce such complexity

into independent mechanic components is limited consequentially, generally unreasonable and hence misleading. Moreover, if any one of the mutually entangled parties is removed from the equation then the state of warfare, possibly to be replaced with another. That is to say that, warfare is essentially a holistic phenomenon, both in terms of global nations involved and in terms of autonomous warfare systems (see [13–15]).

Quantum computation has the potential to significantly affect many areas of human activity, which is especially true for the military area. Quantum computing can impact all the domains of modern warfare. Moreover, the forthcoming quantum revolution will improve sensitivity and efficiency of computing in artificial intelligence, and introduce new capabilities and sharpen modern warfare techniques^[16].

A universal framework developed in this paper to claim national and international viewpoint of war and battles and to express this fundamental observation that conflicts are not isolated but comprise many entangled conflicts at various levels of involvement, occurring simultaneously in the world, is called the entangled warfare category.

2 The Lanchester Equations

In 1916, Frederick William Lanchester proposed on a mathematical model of two fighting forces by a set of ordinary differential equations (ODE)^[17]. The state variables of the ODE represent fighting forces in the battlefield, and each of the ODEs capture the rate of attrition in a certain state variable as a function of the other state variables^[18]. Let $B = B(t) : \mathbb{R} \rightarrow \mathbb{R}$ and $R = R(t) : \mathbb{R} \rightarrow \mathbb{R}$ with their respective initial strength B_0 and R_0 and their corresponding combat-effectiveness coefficients $k_B \in \mathbb{R}^+$ and $k_R \in \mathbb{R}^+$, be state variables denoting the strength of the surviving combatants at time t of the Blue force and the Red force, respectively^[19]. The Legacy Lanchester equations are of the following two basic types:

1) Lanchester square law for aimed-fire:

$$\begin{aligned} \text{Red: } \dot{R} &= -k_R B, \\ \text{Blue: } \dot{B} &= -k_B R, \end{aligned} \tag{1}$$

where overdot denotes time derivative. Observing the state equation $k_R(B_0^2 - B^2) = k_B(R_0^2 - R^2)$, we know that while the attrition rate has a linear effect, the effect of size is quadratic. This is the cause that the aimed-fire model is also called the Square Law. The Square Law underscores the importance of concentration of forces, which is a well-known principle in military tactics.

2) Lanchester linear law for area-unaimed fire:

$$\begin{aligned} \text{Red: } \dot{R} &= -k_{BR} B R, \\ \text{Blue: } \dot{B} &= -k_{RB} R B, \end{aligned} \tag{2}$$

where k_{BR} and k_{RB} denote mixed combat-rate coefficients for Red and Blue forces. Observing the state equation $k_R(B_0 - B) = k_B(R_0 - R)$, we know that the effect of one's fire does not only depend on the strength of its own surviving force but also on the strength of the targets at

the opposing side. If the fire is not aimed, the probability of hitting a target on the other side depends on the number of such targets in a given area.

In all type of battles described in the Legacy Lanchester equations including aimed fire and area fire, both sides apply the same homogeneous forces; the battles are symmetric. Asymmetric engagements occur when the two sides have disparate military power and/or apply different tactics. One such asymmetric battle situation occurs when the strong forces of a state fight the weak forces who apply irregular warfare tactics such as the initial phase of the Russo-Ukrainian War in 2022 and the US launching the War in Afghanistan from 2001 to 2021. If B is the strong force which applies to regular tactics, while R represents the weak force which applies irregular warfare tactics, then the attrition equations^[20,21] are:

$$\begin{aligned} \text{Red: } \dot{R} &= -k_R B, \\ \text{Blue: } \dot{B} &= -k_B \frac{R}{R_0}, \end{aligned} \quad (3)$$

and the state equation is $\frac{k_R}{2}(B_0^2 - B^2) = k_B(R_0^2 - R_0 R)$ which implies a mixture of the two Lanchester laws: the Square Law and the Linear Law. One consider an aimed-fire situation where a homogeneous Blue force is engaged in battle with a heterogeneous Red force comprising n units $R_1, R_2, \dots, R_n$, where $n \in \mathbb{N}$. The n Red units are various in terms of battle-effectiveness and vulnerability. Thus, The Lanchester equations (3) in this case are rewritten as:

$$\begin{aligned} \text{Red: } \dot{R}(t) &= -\lambda_i(t) k_R^i B(t), \\ \text{Blue: } \dot{B}(t) &= -\sum_{i=1}^n k_B^i R(t), \end{aligned} \quad (4)$$

where for all $t \sum_{i=1}^n \lambda_i(t) = 1$, and $i = 1, 2, \dots, n$. In the above Lanchester equations (4), k_R^i denote the kill rate of Blue(R_i) against R_i (Blue). While Red force employs all its n units against Blue force, the latter has to think how to dynamically allocate its fire among its n rivals. In other words, at any given time t in the battle, the Blue force has to choice what fraction $\lambda_i(t)$ of its force to allocate for engaging $R_i(t)$.

In the other way, both linear and quadratic Lanchester-type terms (see [22–27]):

$$\begin{aligned} \text{Red: } \overbrace{\partial_t R^a}^{\text{Red.vecfield}} &= \overbrace{k A_b^a B^b}^{\text{square.Lanchaster}} + \overbrace{k_b F_{cd}^{ab} R^c B^d}^{\text{linear.Lanchaster}} \in M_{\text{Red}}, \\ \text{Blue: } \overbrace{\partial_t B^a}^{\text{Blue.vecfield}} &= \overbrace{\kappa C_b^a R^b}^{\text{square.Lanchaster}} + \overbrace{\kappa_b G_{cd}^{ab} R^c B^d}^{\text{linear.Lanchaster}} \in M_{\text{Blue}}, \end{aligned} \quad (5)$$

where $\partial_t = \frac{\partial}{\partial t}$ and the Red and Blue strength of forces are defined as vectors $R^a = R^a(x, t) \in M_{\text{Red}}$ and $B^a = B^a(y, t) \in M_{\text{Blue}}$, on their respective configuration n -manifold M_{Red} and M_{Blue} with their respective local coordinates $\{x^a\}$ and $\{y^a\}$ for $t = 1, \dots, n$. Therefore, the Lanchester equation including (1) and (2) can be updated to the forms of the Red and Blue vectorfields $\partial_t R^a$ and $\partial_t B^a$, including the following terms (see [23]):

① Lanchester square law: $kA_b^a B^b \in M_{\text{Red}}$ and $\kappa C_b^a R^b \in M_{\text{Blue}}$, where A_b^a and C_b^a represent the battle tensors of the Red side and Blue side.

② Lanchester linear law: $k_b F_{cd}^{ab} R^c B^d \in M_{\text{Red}}$ and $\kappa_b G_{cd}^{ab} R^c B^d \in M_{\text{Blue}}$, where the 4th-order tensors F_{cd}^{ab} and G_{cd}^{ab} represent strategic, tactical and operational capabilities of the Red and Blue sides.

Under the asymmetric engagement situation^[28], the pair of equations (5) are rewritten as in term of equation (4):

$$\begin{aligned} \text{Red: } \overbrace{\partial_t R^r}^{\text{Red.vecfield}} &= \overbrace{\lambda_i k_R^i A_b^r B^b}^{\text{square.Lanchester}} + \overbrace{\lambda_i k_{BR}^i F_{cd}^{rb} R^c B^d}^{\text{linear.Lanchester}} \in M_{\text{Red}}, \\ \text{Blue: } \overbrace{\partial_t B^s}^{\text{Blue.vecfield}} &= \sum_{i=1}^v \overbrace{k_B^i C_b^s R^b}^{\text{square.Lanchester}} + \sum_{i=1}^v \overbrace{k_{RB}^i G_{cd}^{sb} R^c B^d}^{\text{linear.Lanchester}} \in M_{\text{Blue}}, \end{aligned} \quad (6)$$

where for all $t \sum_{i=1}^v \lambda_i(t) = 1$ ($i \in \mathbb{N}$), and the Red and Blue strength of forces are redefined as vectors $R^r = R^r(x, t) \in M_{\text{Red}}$ and $B^s = B^s(y, t) \in M_{\text{Blue}}$, defined on their respective configuration u -manifold M_{Red} and v -manifold M_{Blue} with their respective local coordinates $\{x^r\}$ for $r = 1, \dots, u$ and $\{y^s\}$ for $s = 1, \dots, v$.

3 Entangled Manifold Categories

The basic algebraic operator which we use to represent entangled fusion in the view of manifold, is the ordinary tensor product and its categorical abstraction. Specifically, the Kronecker tensor product, usually denoted by “ $\otimes$ ” and based on the Cartesian product, is universal in mathematics.

Implicitly, the pair of equations (6) use tensor products between all included tensor fields. Explicitly, these Lanchester equations (6) are rewritten as:

$$\begin{aligned} \text{Red: } \overbrace{\partial_t R^r}^{\text{Red.vecfield}} &= \overbrace{\lambda_i \otimes k_R \otimes A_b^r \otimes B^b}^{\text{square.Lanchester}} + \overbrace{\lambda_i \otimes k_{BR} \otimes F_{cd}^{rb} \otimes R^c \otimes B^d}^{\text{linear.Lanchester}} \in M_{\text{Red}}, \\ \text{Blue: } \overbrace{\partial_t B^s}^{\text{Blue.vecfield}} &= \sum_{i=1}^v \overbrace{k_B^i \otimes C_b^s \otimes R^b}^{\text{square.Lanchester}} + \sum_{i=1}^v \overbrace{k_{RB}^i \otimes G_{cd}^{sb} \otimes R^c \otimes B^d}^{\text{linear.Lanchester}} \in M_{\text{Blue}}. \end{aligned} \quad (7)$$

The tensor product $\otimes$ sticks together various tensor fields into inseparable terms, representing inseparable warfighting actions.

Similarly, in quantum computation^[6], a compound system $|\psi\rangle_1 \otimes |\psi\rangle_2$ consists of two entangled wave-functions, $|\psi\rangle_1$ and $|\psi\rangle_2$, which can be physically far-apart but still have their maximally-entangled Bell-states: $|\psi\rangle_{12} = |\psi\rangle_1 \otimes |\psi\rangle_2$, which exists in the tensor product $\mathcal{H}_1 \otimes \mathcal{H}_2$ of their respective Hilbert spaces of states, $\mathcal{H}_1$ and $\mathcal{H}_2$. Therefore, the tensor product $\otimes$ ^[27] acts both locally, on the wave-functions, as $\otimes : |\psi\rangle_1 \times |\psi\rangle_2 \rightarrow |\psi\rangle_1 \otimes |\psi\rangle_2$, where $\otimes : |\psi\rangle_1 \times |\psi\rangle_2$ is the Cartesian product of the wave-functions, and globally, on their respective Hilbert spaces, as $\otimes : \mathcal{H}_1 \times \mathcal{H}_2 \rightarrow \mathcal{H}_1 \otimes \mathcal{H}_2$, where $\mathcal{H}_1 \times \mathcal{H}_2$ is the Cartesian product of the Hilbert spaces.

Generalization to any number of wave-functions (and their respective Hilbert spaces of states) is straightforward.

Supposing that there are a pair of complex systems: A and B , one will fuse them into one system, namely a single entangled $[A \otimes B]$. The most natural framework for such a combination is a tensor category^[29].

For simplicity, one define a symmetric monoidal tensor (SMT) category $\mathcal{C}$, using the tensor functor $\otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$ that acts on a pair $(\text{Ob}(\mathcal{C}), \text{Mor}(\mathcal{C}))$ of general objects of $\mathcal{C} : A, B, \dots \in \text{Ob}(\mathcal{C})$ and general morphisms (arrows) between objects of $\mathcal{C}$, where $f : A \rightarrow B, g : B \rightarrow C, \dots \in \text{Mor}(\mathcal{C})$, and $\mathcal{C} \times \mathcal{C}$ means the Cartesian product of the category $\mathcal{C}$ with itself. The process of the tensor functor $\otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$ is given on objects and morphisms of $\mathcal{C}$ by:

$$\begin{aligned} \text{Ob}(\mathcal{C}) : \{A, B\} &\mapsto A \otimes B \equiv B \otimes A, \\ \text{Mor}(\mathcal{C}) : \{A \xrightarrow{f} B, C \xrightarrow{g} D\} &\mapsto A \otimes C \xrightarrow{f \otimes g} B \otimes D. \end{aligned} \quad (8)$$

The key feature of the tensor functor $\otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$ is the commutative bifunctionality. That is to say, the order of two operations f and g applied to both different subsystem does not matter (see [30]). Briefly, for any quadruple (A_1, A_2, B_1, B_2) of systems with the corresponding quadruple of processes (f_1, f_2, g_1, g_2) between them, we have the following bi-compositional rule:

$$(g_1 \otimes g_2) \circ (f_1 \otimes f_2) = (g_1 \circ f_1) \otimes (g_2 \circ f_2), \quad (9)$$

where “ $\circ$ ” means a temporal composition in which the output of the first process $f : A \rightarrow B$ becomes the input to the second process $g : B \rightarrow C$ etc., formally defined as: $A \xrightarrow{f} B \xrightarrow{g} C = Ag \circ fc$.

This rule of any SMT category $\mathcal{C}$ shows that any combinations of serial and parallel compositions of systems are commutative regardless of their order, due to the symmetry of the tensor functor $\otimes$, in spite of the time-asymmetry of the serial composition “ $\circ$ ”.

In two Hilbert spaces, $\mathcal{H}_A$ and $\mathcal{H}_B$, are objects of the SMT category $\mathcal{HILB}_2$ with the quantum tensor functor, $\otimes : \mathcal{HILB}_2 \times \mathcal{HILB}_2 \rightarrow \mathcal{HILB}_2$, given by:

$$\begin{aligned} \text{Ob}(\mathcal{HILB}_2) : \{\mathcal{H}_A, \mathcal{H}_B\} &\mapsto \mathcal{H}_A \otimes \mathcal{H}_B \equiv \mathcal{H}_B \otimes \mathcal{H}_A, \\ \text{Mor}(\mathcal{HILB}_2) : \{\mathcal{H}_A \xrightarrow{f} \mathcal{H}_B, \mathcal{H}_C \xrightarrow{g} \mathcal{H}_D\} &\mapsto \mathcal{H}_A \otimes \mathcal{H}_C \xrightarrow{f \otimes g} \mathcal{H}_B \otimes \mathcal{H}_D, \end{aligned} \quad (10)$$

where the tensor product of Hilbert spaces:

$$\mathcal{H}_A \otimes \mathcal{H}_B = \left\{ \sum_{k,j} \omega_{kj} \cdot |k\rangle_A \otimes |j\rangle_B, \omega_{kj} \in \mathbb{C}, \forall k, j \right\}, \quad (11)$$

includes the entangled $[A \otimes B]$ Bell-state^[31]:

$$|\psi\rangle_{AB} \equiv |\psi\rangle_A \otimes |\psi\rangle_B = \sum_{k,j} \omega_{kj} \cdot |k\rangle_A \otimes |j\rangle_B \in \mathcal{H}_A \otimes \mathcal{H}_B. \quad (12)$$

Generalization of the 2D SMT category $\mathcal{HILB}_2$ to a large-scale SMT category $\mathcal{HILB}_N$ is defined by the high-dimensional tensor functor, $\otimes : \mathcal{HILB}_N \times \mathcal{HILB}_N \rightarrow \mathcal{HILB}_N$, given by:

$$\begin{aligned} \text{Ob}(\mathcal{HILB}_N) : \{\mathcal{H}_{ij}, \mathcal{H}_{km}\} &\mapsto \mathcal{H}_{ij} \otimes \mathcal{H}_{km} \equiv \mathcal{H}_{km} \otimes \mathcal{H}_{ij}, (i, j, k, m \in \mathbb{N}), \\ \text{Mor}(\mathcal{HILB}_N) : \{\mathcal{H}_{ij} \xrightarrow{f} \mathcal{H}_{km}, \mathcal{H}_{pq} \xrightarrow{g} \mathcal{H}_{rs}\} &\mapsto \mathcal{H}_{ij} \otimes \mathcal{H}_{km} f \otimes g \mathcal{H}_{pq} \otimes \mathcal{H}_{rs}, \end{aligned} \quad (13)$$

with objects given by the following matrix of Hilbert spaces:

$$(\mathcal{H})_{NN} = \begin{bmatrix} \mathcal{H}_{11} & \otimes & \mathcal{H}_{12} & \otimes & \mathcal{H}_{13} & \otimes & \cdots & \otimes & \cdots & \mathcal{H}_{1N} \\ \otimes & \cdots & \otimes & \cdots & \otimes & \cdots & \cdots & \cdots & \cdots & \otimes \\ \mathcal{H}_{21} & \otimes & \mathcal{H}_{22} & \otimes & \mathcal{H}_{23} & \otimes & \cdots & \otimes & \cdots & \mathcal{H}_{2N} \\ \otimes & \cdots & \otimes & \cdots & \otimes & \cdots & \cdots & \cdots & \cdots & \otimes \\ \mathcal{H}_{31} & \otimes & \mathcal{H}_{32} & \otimes & \mathcal{H}_{33} & \otimes & \cdots & \otimes & \cdots & \mathcal{H}_{3N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathcal{H}_{N1} & \otimes & \mathcal{H}_{N2} & \otimes & \mathcal{H}_{N3} & \otimes & \cdots & \otimes & \cdots & \mathcal{H}_{NN} \end{bmatrix} \in \mathcal{HILB}_N. \quad (14)$$

Every element in the matrix (14) represents the Hilbert state-space of a single wave-function with its own quantum state: $|\psi\rangle_{kk} \in \mathcal{H}_{kk}$ (for $k = 1, \dots, N$) so that all quantum states form the corresponding complex-valued matrix, in which every tensor product $\otimes_{kj} \in (|\psi\rangle)_{NN}$ has its associated weight matrix $(\omega)_{kj} \in \mathbb{C}$:

$$(|\psi\rangle)_{NN} = \begin{bmatrix} |\psi\rangle_{11} & \otimes & |\psi\rangle_{12} & \otimes & |\psi\rangle_{13} & \otimes & \cdots & \otimes & \cdots & |\psi\rangle_{1N} \\ \otimes & \cdots & \otimes & \cdots & \otimes & \cdots & \cdots & \cdots & \cdots & \otimes \\ |\psi\rangle_{21} & \otimes & |\psi\rangle_{22} & \otimes & |\psi\rangle_{23} & \otimes & \cdots & \otimes & \cdots & |\psi\rangle_{2N} \\ \otimes & \cdots & \otimes & \cdots & \otimes & \cdots & \cdots & \cdots & \cdots & \otimes \\ |\psi\rangle_{31} & \otimes & |\psi\rangle_{32} & \otimes & |\psi\rangle_{33} & \otimes & \cdots & \otimes & \cdots & |\psi\rangle_{3N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ |\psi\rangle_{N1} & \otimes & |\psi\rangle_{N2} & \otimes & |\psi\rangle_{N3} & \otimes & \cdots & \otimes & \cdots & |\psi\rangle_{NN} \end{bmatrix} \in (\mathcal{H})_{NN}. \quad (15)$$

4 The Entangled Warfare Category

Now, inspired by the model (14)~(15), we can introduce our main result, the high-dimensional warfare-category $\mathcal{WAR}$ -as a tensor entanglement of all existing Red and Blue battle-manifolds^[23]:

$$\mathcal{WAR} = M_{\text{Red}}^{NN} \otimes M_{\text{Blue}}^{NN}. \quad (16)$$

Specifically, the objects and morphisms of the $\mathcal{WAR}$ -category are defined by the action of the high-dimensional warfare tensoring functor, $\otimes : \mathcal{WARfare} \times \mathcal{WARfare} \rightarrow \mathcal{WARfare}$, given by:

$$\begin{aligned} \text{Ob}(\mathcal{WAR}) : \{[M_{\text{Red}}^{ij} \otimes M_{\text{Blue}}^{ij}], [M_{\text{Red}}^{km} \otimes M_{\text{Blue}}^{km}]\} &\mapsto [M_{\text{Red}}^{ij} \otimes M_{\text{Blue}}^{ij}] \otimes [M_{\text{Red}}^{km} \otimes M_{\text{Blue}}^{km}], \\ \text{Mor}(\mathcal{WAR}) : \{[M_{\text{Red}}^{ij} \otimes M_{\text{Blue}}^{ij}] \xrightarrow{f} [M_{\text{Red}}^{km} \otimes M_{\text{Blue}}^{km}], [M_{\text{Red}}^{pq} \otimes M_{\text{Blue}}^{pq}] \xrightarrow{g} [M_{\text{Red}}^{rs} \otimes M_{\text{Blue}}^{rs}]\} &\mapsto \\ [M_{\text{Red}}^{ij} \otimes M_{\text{Blue}}^{ij}] \otimes [M_{\text{Red}}^{km} \otimes M_{\text{Blue}}^{km}] f \otimes g [M_{\text{Red}}^{pq} \otimes M_{\text{Blue}}^{pq}] \otimes [M_{\text{Red}}^{rs} \otimes M_{\text{Blue}}^{rs}], \end{aligned} \quad (17)$$

with all indices going from $1, \dots, N$.

In the warfare-category: $\mathcal{WAR} = M_{\text{Red}}^{NN} \otimes M_{\text{Blue}}^{NN}$, the sub-objects M_{Red}^{NN} are represented by the following $N \times N$ matrices of Red and Blue battle-manifolds, respectively:

$$M_{\text{Red}}^{NN} = \begin{bmatrix} M_{\text{Red}}^{11} & \otimes & M_{\text{Red}}^{12} & \otimes & M_{\text{Red}}^{13} & \otimes & \dots & \otimes & \dots & M_{\text{Red}}^{1N} \\ \otimes & \dots & \otimes & \dots & \otimes & \dots & \dots & \dots & \dots & \otimes \\ M_{\text{Red}}^{21} & \otimes & M_{\text{Red}}^{22} & \otimes & M_{\text{Red}}^{23} & \otimes & \dots & \otimes & \dots & M_{\text{Red}}^{2N} \\ \otimes & \dots & \otimes & \dots & \otimes & \dots & \dots & \dots & \dots & \otimes \\ M_{\text{Red}}^{31} & \otimes & M_{\text{Red}}^{32} & \otimes & M_{\text{Red}}^{33} & \otimes & \dots & \otimes & \dots & M_{\text{Red}}^{3N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M_{\text{Red}}^{N1} & \otimes & M_{\text{Red}}^{N2} & \otimes & M_{\text{Red}}^{N3} & \otimes & \dots & \otimes & \dots & M_{\text{Red}}^{NN} \end{bmatrix} \in \mathcal{WAR}, \quad (18)$$

and

$$M_{\text{Blue}}^{NN} = \begin{bmatrix} M_{\text{Blue}}^{11} & \otimes & M_{\text{Blue}}^{12} & \otimes & M_{\text{Blue}}^{13} & \otimes & \dots & \otimes & \dots & M_{\text{Blue}}^{1N} \\ \otimes & \dots & \otimes & \dots & \otimes & \dots & \dots & \dots & \dots & \otimes \\ M_{\text{Blue}}^{21} & \otimes & M_{\text{Blue}}^{22} & \otimes & M_{\text{Blue}}^{23} & \otimes & \dots & \otimes & \dots & M_{\text{Blue}}^{2N} \\ \otimes & \dots & \otimes & \dots & \otimes & \dots & \dots & \dots & \dots & \otimes \\ M_{\text{Blue}}^{31} & \otimes & M_{\text{Blue}}^{32} & \otimes & M_{\text{Blue}}^{33} & \otimes & \dots & \otimes & \dots & M_{\text{Blue}}^{3N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M_{\text{Blue}}^{N1} & \otimes & M_{\text{Blue}}^{N2} & \otimes & M_{\text{Blue}}^{N3} & \otimes & \dots & \otimes & \dots & M_{\text{Blue}}^{NN} \end{bmatrix} \in \mathcal{WAR}. \quad (19)$$

In the matrices of battle-manifolds (18)~(19) the corresponding Red and Blue vectorfields, R_{ik}^a and B_{ik}^a ($i, k = 1, 2, \dots, N$), are acting in their respective operation-manifolds:

$$R_{NN}^a = R_{NN}^a(\mathbf{x}, t) = \begin{bmatrix} R_{11}^a & \otimes & R_{12}^a & \otimes & R_{13}^a & \otimes & \dots & \otimes & \dots & R_{1N}^a \\ \otimes & \dots & \otimes & \dots & \otimes & \dots & \dots & \dots & \dots & \otimes \\ R_{21}^a & \otimes & R_{22}^a & \otimes & R_{23}^a & \otimes & \dots & \otimes & \dots & R_{2N}^a \\ \otimes & \dots & \otimes & \dots & \otimes & \dots & \dots & \dots & \dots & \otimes \\ R_{31}^a & \otimes & R_{32}^a & \otimes & R_{33}^a & \otimes & \dots & \otimes & \dots & R_{3N}^a \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ R_{N1}^a & \otimes & R_{N2}^a & \otimes & R_{N3}^a & \otimes & \dots & \otimes & \dots & R_{NN}^a \end{bmatrix}, \quad (20)$$

and

$$B_{NN}^a = B_{NN}^a(\mathbf{x}, t) = \begin{bmatrix} B_{11}^a & \otimes & B_{12}^a & \otimes & B_{13}^a & \otimes & \dots & \otimes & \dots & B_{1N}^a \\ \otimes & \dots & \otimes & \dots & \otimes & \dots & \dots & \dots & \dots & \otimes \\ B_{21}^a & \otimes & B_{22}^a & \otimes & B_{23}^a & \otimes & \dots & \otimes & \dots & B_{2N}^a \\ \otimes & \dots & \otimes & \dots & \otimes & \dots & \dots & \dots & \dots & \otimes \\ B_{31}^a & \otimes & B_{32}^a & \otimes & B_{33}^a & \otimes & \dots & \otimes & \dots & B_{3N}^a \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B_{N1}^a & \otimes & B_{N2}^a & \otimes & B_{N3}^a & \otimes & \dots & \otimes & \dots & B_{NN}^a \end{bmatrix}. \quad (21)$$

In the matrices (20)~(21), the Red and Blue vectorfields, R_{ik}^a and B_{ik}^a ($a = 1, \dots, N$), are governed by the high-dimensional generalization^[27] of the standard Lanchester-combat equations (6):

$$\begin{aligned}
 \text{Red: } \overbrace{\partial_t R^r}^{\text{Red.vecfield}} &= \overbrace{\lambda_i k_R^i A_b^r B^b}^{\text{square.Lanchaster}} + \overbrace{\lambda_i k_{BR}^i F_{cd}^{rb} R^c B^d}^{\text{linear.Lanchaster}} + \overbrace{R^b \mathcal{L}_R \mathcal{N}_b^r}^{\text{Lie-dragging}} + \overbrace{[R^r, B^s]}^{\text{war.symmetry}} + \overbrace{\delta R^r (H - L)}^{\text{delta.strikes}}, \\
 \text{Blue: } \overbrace{\partial_t B^s}^{\text{Blue.vecfield}} &= \sum_{i=1}^v \overbrace{k_B^i C_b^s R^b}^{\text{square.Lanchaster}} + \sum_{i=1}^v \overbrace{k_{RB}^i G_{cd}^{sb} R^c B^d}^{\text{linear.Lanchaster}} + \overbrace{B^b \mathcal{L}_B \mathcal{M}_b^s}^{\text{Lie-dragging}} + \overbrace{[B^s, R^r]}^{\text{war.symmetry}} + \overbrace{\delta B^s (H - L)}^{\text{delta.strikes}},
 \end{aligned} \tag{22}$$

where $R^b \mathcal{L}_R \mathcal{N}_b^r \in M_{\text{Red}}$ and $B^b \mathcal{L}_B \mathcal{M}_b^s \in M_{\text{Blue}}$ are entropic Lie-dragging terms; $[R^s, B^t] \in M_{\text{Red}}$ and $[B^t, R^s] \in M_{\text{Blue}}$ are entropic Red-Blue commutators; $\delta R^r (H - L)$ and $\delta B^s (H - L)$ are Hamilton-Langevin delta strikes (see [24–27]).

5 Discussion

In summary, by specializing the general tensor functor $\otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$ into the warfare tensor functor, $\otimes : \mathcal{WAR} \times \mathcal{WAR} \rightarrow \mathcal{WAR}$, we have developed the Holistic Warfare model as the high-dimensional entangled warfare category:

$$\mathcal{WAR} \equiv [\text{Ob}(\mathcal{WAR}), \text{Mor}(\mathcal{WAR})] = M_{\text{Red}}^{NN} \otimes M_{\text{Blue}}^{NN}, \tag{23}$$

where the two warfighting sides, Red and Blue, include all involved Red-Blue battle-manifolds (18)~(19), in which the action of the corresponding Red-Blue vectorfields (20)~(21) is happening according to the high-dimensional battle Equations(22).

Our Holistic Warfare model, the $\mathcal{WAR}$ -category, has the following key characteristics^[27]:

- 1) Warfare includes all interaction battlefields occurring in an abstract warfare scenario, represented by the entangled fusion, $M_{\text{Red}}^{NN} \times M_{\text{Blue}}^{NN}$ of the Red and Blue matrices of the battle-manifolds (18)~(19);
- 2) Each individual battlefield is represented by the entangled fusion, $M_{\text{Red}} \otimes M_{\text{Blue}}$, of the individual Red and Blue battle-manifolds, M_{Red} and M_{Blue} ;
- 3) Each individual battlefield, $M_{\text{Red}} \otimes M_{\text{Blue}}$, involves the process of the corresponding Red-Blue vectorfields (20)~(21);
- 4) The process of all included Red-Blue vectorfields is governed by the high-dimensional battle Equations (22).

Different from^[27], the Holistic Warfare model has generalized the symmetric conflict scenario to the asymmetric conflict scenario (22). The functoriality of the warfare tensor functor, $\otimes : \mathcal{WAR} \times \mathcal{WAR} \rightarrow \mathcal{WAR}$, can be extended to the case of two main warfighting conflicts, $\mathcal{WAR}_1$ and $\mathcal{WAR}_2$, simultaneously occupying two different parts of the globe and producing the composite output $\mathcal{WAR}_{12}$, as: $\otimes : \mathcal{WAR}_1 \times \mathcal{WAR}_2 \rightarrow \mathcal{WAR}_{12}$. The objects and morphisms of the composite category $\mathcal{WAR}_{12}$ are given by the combined battle-manifolds for the

Red and Blue warfighting sides, $M_{R_{12}}^{NN} \otimes M_{B_{12}}^{NN}$, as:

$$\begin{aligned} \text{Ob}(\mathcal{WAR}_{12}) : \{[M_{R_{12}}^{ij} \otimes M_{B_{12}}^{ij}], [M_{R_{12}}^{km} \otimes M_{B_{12}}^{km}]\} &\mapsto [M_{R_{12}}^{ij} \otimes M_{B_{12}}^{ij}] \otimes [M_{R_{12}}^{km} \otimes M_{B_{12}}^{km}], \\ \text{Mor}(\mathcal{WAR}) : \{[M_{R_{12}}^{ij} \otimes M_{B_{12}}^{ij}] \xrightarrow{f} [M_{R_{12}}^{km} \otimes M_{B_{12}}^{km}], [M_{R_{12}}^{pq} \otimes M_{B_{12}}^{pq}] \xrightarrow{g} [M_{R_{12}}^{rs} \otimes M_{B_{12}}^{rs}]\} &\mapsto \\ [M_{R_{12}}^{ij} \otimes M_{B_{12}}^{ij}] \otimes [M_{R_{12}}^{km} \otimes M_{B_{12}}^{km}] f \otimes g [M_{R_{12}}^{pq} \otimes M_{B_{12}}^{pq}] \otimes [M_{R_{12}}^{rs} \otimes M_{B_{12}}^{rs}], \end{aligned}$$

with the subsequent generalization of the manifold matrices (18)~(19), vectorfields (20)~(21) and battle Equations (22).

The generalized warfare category $\mathcal{WAR}_{12}$ shows that whenever there are two apparently unrelated warfighting conflicts in different parts of the world (e.g., on different continents), ultimately they would be entangled into the same global warfare. That is to say, even though we have two geographically-different conflicts, the proposed framework suggests that they can be linked through entangled fusion.

The concept of Holistic Warfare is illustrated in this section and we have shown a practical approach to the global modeling of warfare. The framework proposed here has clearly allowed for the capture of the complexity of the Holistic Warfare through the use of the concept of entanglement.

6 Conclusion

In this paper we have proposed a Holistic Warfare model under the asymmetric conflict scenario, represented by the high-dimensional entangled category $\mathcal{WAR}$. The metaphysics of this Holistic Warfare model is “entangled states fusion”, the macroscopic entanglement concept, inspired by high-dimensional quantum computation in which any number of entangled wave-functions can be highly-correlated. In this “entangled view”, the warfare is essentially a global phenomenon, thus if any one of the mutually entangled conflict sides is removed from the equation, the conflict, in its present form, instantly stops, though possibly being instantly replaced by another global warfare situation.

This description of conflicts within our global entangled model thus describes a different identity to distinct time series within a larger surrounding network of conflict, as different relevant sides enter or exit different states of conflict or cooperation with other sides. We believe this more effectively captures the characteristics in which past and present geopolitical conflict manifests; in particular, our model here is intended to represent war and battle more from a national perspective, rather than from the view of isolated individual battles temporally linearly arranged.

The formal Holistic Warfare framework developed in this paper shows this main topic of many simultaneously interrelated battlefields, each of them defined by its own battle-manifold and occurring simultaneously and worldwide which is called the entangled category $\mathcal{WAR}$. The process of the $\mathcal{WAR}$ -category is formally defined using the warfare tensor functor $\otimes : \mathcal{WAR} \times \mathcal{WAR} \rightarrow \mathcal{WAR}$.

In this paper, we have considered recent conflict scenarios which are not regular in the sense

that they are profoundly asymmetric. The research works will increasingly rely on data and information, because the modern warfare are more and more affected by the behavior of civilians and they may involve more than two adversaries.

The future warfare and battle situation may include several characteristics, such as multi-lateral conflicts^[32], cognitive warfare, biological warfare and unmanned operation so on, which lend themselves to new mathematical modeling.

Quantum computing and artificial intelligence (see [33,34]) are in the emerging area of technologies that change the future warfare with the potential to be disruptive. Those changes will require the development of new strategies, tactics and policies, assessment of cooperations and conflicts among different parties, which have strategic, worldwide and long-term impacts.

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