

## A Novel Multi-Attribute Decision-Making Method Based on Probabilistic Hesitant Fuzzy Soft Set and Its Application

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**Abstract** The probabilistic hesitant fuzzy multi-attribute group decision-making method introduces probability and hesitation into decision-making problems at the same time, which can improve the reliability and accuracy of decision-making results, and has become a research hotspots in recent years. However, there are still many problems, such as overly complex calculations and difficulty in obtaining probability data. Based on these, the paper proposes a multi-attribute group decision-making model based on probability hesitant fuzzy soft sets. Firstly, the definition of probabilistic hesitant fuzzy soft set is given. Then, based on soft set theory and probabilistic hesitant fuzzy set, the similarity measure of probabilistic hesitant fuzzy soft set is proposed, and the two measures are further combined. Finally, it is applied to the construction of multi-attribute group decision-making model, and the effectiveness and rationality of the model are verified by an example. The example shows that the new similarity calculation formula and algorithm model in this paper have higher accuracy, and the calculation process is more simple, it provides a feasible method for multi-attribute group decision making problems.

**Keywords** probabilistic hesitant fuzzy soft sets; similarity; multi-attribute group decision-making model

## 1 Introduction

Multi-attribute decision-making based on soft sets refers to the decision-making problem of selecting the optimal alternative or ranking the alternatives when considering multiple attributes. At present, it is widely used in medical diagnosis<sup>[1, 2]</sup>, environmental risk

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Received September 2, 2023, accepted January 8, 2024

Supported by 2023 Henan Provincial Department of Science and Technology Key R&D and Promotion Special Project (Soft Science Research) (232400411049), Henan Province Science and Technology Research and Development Plan Joint Fund (Industry) Project (225101610054)

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identification<sup>[3]</sup>, teaching quality assessment<sup>[4]</sup>, supplier selection<sup>[5]</sup> and other fields<sup>[6, 7]</sup>. It can be seen that the main problems solved by multi-attribute decision-making are evaluation and selection<sup>[8, 9]</sup>.

Since Zadeh<sup>[10]</sup> first proposed fuzzy sets in 1965, fuzzy sets and its extensions have been widely used in scientific decision making. With the appearance of complex decision-making problems, we can find that fuzzy sets can not fully represent the evaluation information. In order to overcome the problem that experts in decision-making are hesitant between several possible values, Torra<sup>[11]</sup> proposed hesitant fuzzy sets and Mockor and Hynar<sup>[12]</sup> gave its mathematical expression. However, with the rapid development of hesitant fuzzy sets theory, its drawbacks have gradually become prominent. Here we take the following example as an illustration: There are three decision makers (DMs) who rated a scheme as 0.2 under a certain attribute and two DMs rated 0.3. Then the hesitant fuzzy set is expressed as  $\{0.2, 0.3\}$ , which is obviously unreasonable and one decision makers' evaluation information is ignored. To solve this problem, Zhu and Xu<sup>[13]</sup> proposed probability hesitant fuzzy sets (PHFS) by adding probability information to the membership degree. The above example can be expressed by probability hesitant fuzzy sets as  $\{0.2|0.6, 0.3|0.4\}$ . Obviously, this expression solves the problem of the same membership probability of hesitant fuzzy sets and represents the evaluation information of decision-makers more completely. Subsequently, In 2017, a hesitant fuzzy number with probabilities called the hesitant probabilistic fuzzy number was proposed, and its score function, deviation function, comparison laws, and its basic operations were constructed. Subsequently, more in-depth research around it was gradually launched<sup>[14, 15]</sup>. Yang and Xu<sup>[16]</sup> defined an extended probabilistic hesitant I-fuzzy set (PHIFS), and proposed a simple and effective method based on weighted area index to measure fuzzy information in decision-making problems. Zhou and Xu<sup>[17]</sup> proposed the probabilistic hesitant fuzzy preference relations (PHFPRs) based on the probabilistic hesitant fuzzy element (PHFE). Liao, et al.<sup>[18]</sup> introduced the Extended TODIM Method Based on Cumulative Prospect Theory (CPT) for probabilistic hesitant fuzzy multiple attributes group decision-making (MAGDM).

To solve the uncertainty problem in modeling, Molodtsov<sup>[19]</sup> proposed a completely general mathematical tool - soft sets in 1999. A soft set is a two-tuple consisting of a parameter set and a set-valued mapping on the power set to the universe of discourse (ie, the set of objects). In other words, a soft set is a parameterized family of subsets of a given universe, which can be understood as "a collection of sets divided by parameters". This feature makes the concept of soft set cover a wide range, and it is flexible and applicable. In addition, the definition of soft sets not only involves the object universe, but also introduces the parameter space related to the object universe, which makes soft sets more informative compared to other uncertain theories such as fuzzy sets or rough sets. Therefore, it has an important positive role and influence in the field of uncertain information processing and analysis. At present, many scholars have studied the related theories of soft sets, mainly including the theoretical development of soft sets themselves and the cross-integration of soft sets and other uncertainty theories. Yang and Guo<sup>[20]</sup> proposed soft-set relational mapping and inverse soft-set relational mapping, and then discussed the related properties. Based on the study of Babitha and Sunil<sup>[21]</sup>, Park, et al.<sup>[22]</sup> further studied the relation of equivalent soft sets, obtained the soft analogy of many results

about general equivalence relation and partition. Demirtas, et al.<sup>[23]</sup> defined N-polar soft sets and some properties and several theoretical operations of N-polar soft sets are given. Soft set theory has great potential in many applications. Ma, et al.<sup>[24]</sup> proposed a new efficient decision algorithm for interval-valued fuzzy soft sets, the proposed algorithm involves a relatively small amount of calculation, and considers the added objects, which has higher scalability for large-scale data sets. Based on the soft rough set and similarity proposed by Feng, et al.<sup>[25]</sup>, EL Bably, et al.<sup>[26]</sup> defined the soft rough approximation, and used the soft rough approximation and soft rough topology to get the population most susceptible to COVID-19.

The rest of this paper is organized as follows. Section 2 reviews some basic definitions, operations and similarity calculations of probabilistic hesitant fuzzy sets and soft sets, then gives the basic definition of probabilistic hesitant fuzzy soft sets. In Section 3, the similarity measurement of probability hesitant fuzzy soft sets based on the similarity of soft sets and the distance measure of probability hesitant fuzzy sets are proposed and combined. Then, a multi-attribute group decision-making model based on probability hesitant fuzzy soft set is constructed. In Section 4, a case example for probabilistic hesitant fuzzy soft sets is given to illustrate the proposed model. The conclusions are presented in Section 5.

## 2 Preliminaries and Basic Definition

In this section, we review the basic knowledge about probabilistic hesitant fuzzy sets and soft sets, then introduce some preliminaries used throughout the paper.

### 2.1 Probabilistic Hesitant Fuzzy Sets

**Definition 2.1**<sup>[27]</sup> Let  $R$  be a fixed set, then a PHFS on  $R$  is expressed by a mathematical symbol

$$H_p = \{\langle x, \bar{h}_x(\gamma_l | p_l) \mid x \in R \rangle\}, \quad (1)$$

where  $\bar{h}_x(\gamma_l | p_l)$  is a set of some elements  $\gamma_l | p_l$  denoting the probabilistic membership degrees of the element  $x$  to the set  $H_p$ ,  $p_l$  is the occurrence probability of the membership degree  $\gamma_l$ ,  $\#\bar{h}_x$  is the number of possible elements in  $\bar{h}_x(\gamma_l | p_l)$ ,  $0 \leq \gamma_l \leq 1$ ,  $0 \leq p_l \leq 1$  and 1) if  $\sum_{l=1}^{\#\bar{h}_x} p_l = 1$ , PHFS is standard; 2) if  $\sum_{l=1}^{\#\bar{h}_x} p_l \neq 1$ , PHFS is not standard, we can use  $\tilde{p}_l = \frac{p_l}{\sum_{l=1}^{\#\bar{h}_x} p_l}$  to standardize the probability.

For convenience, we call  $\bar{h}_x(\gamma_l | p_l)$  (or  $\bar{h}(\gamma_l | p_l)$ ) a PHFE, and  $H_p$  the set of all PHFSs. The corresponding score function, deviation function, comparison laws are as follows.

**Definition 2.2**<sup>[27]</sup> For a PHFE  $\bar{h}(\gamma_l | p_l)$  where  $l = 1, 2, \dots, \#\bar{h}$  and  $\sum_{l=1}^{\#\bar{h}_x} p_l = 1$ ,  $s(\bar{h}) = \sum_{l=1}^{\#\bar{h}_x} \gamma_l p_l$  is called score function of  $\bar{h}(\gamma_l | p_l)$ .

**Definition 2.3**<sup>[28]</sup> For a PHFE  $\bar{h}(\gamma_l | p_l)$  where  $l = 1, 2, \dots, \#\bar{h}$  and  $\sum_{l=1}^{\#\bar{h}_x} p_l = 1$ ,  $d(\bar{h}) = \sum_{l=1}^{\#\bar{h}_x} (\gamma_l - s(\bar{h}))^2 p_l$  is called the deviation function of  $\bar{h}(\gamma_l | p_l)$ .

The score and deviation functions are similar to the expectation and variance of random variable, respectively, and thus, the comparison laws of two PHFEs,  $\bar{h}_1$  and  $\bar{h}_2$ , can be presented as: If  $s(\bar{h}_1) > s(\bar{h}_2)$  then  $\bar{h}_1 > \bar{h}_2$ ; if  $s(\bar{h}_1) = s(\bar{h}_2)$  then 1) if  $d(\bar{h}_1) > d(\bar{h}_2)$  then  $\bar{h}_1 < \bar{h}_2$ ; 2) if  $d(\bar{h}_1) = d(\bar{h}_2)$  then  $\bar{h}_1 = \bar{h}_2$ ; and 3) if  $d(\bar{h}_1) < d(\bar{h}_2)$  then  $\bar{h}_1 > \bar{h}_2$ .

Obviously, the PHFE is reduced to the HFE when all probabilities are equal, which shows that the PHFE is a generalized hesitant fuzzy element. The PHFE is more readily used to

present the uncertain elements than the HFE by adding the occurrence probability.

**Definition 2.4**<sup>[28]</sup> Let  $\bar{h}(\gamma_l | p_l)$ ,  $\bar{h}_1(\gamma_t | p_t)$ ,  $\bar{h}_2(\gamma_k | p_k)$  be three PHFEs,  $l = 1, 2, \dots, \#\bar{h}$ ,  $t = 1, 2, \dots, \#\bar{h}_1$ ,  $k = 1, 2, \dots, \#\bar{h}_2$ ,  $\sum_{l=1}^{\#\bar{h}} p_l = 1$ ,  $\sum_{t=1}^{\#\bar{h}_1} p_t = 1$ ,  $\sum_{k=1}^{\#\bar{h}_2} p_k = 1$ , and  $\lambda > 0$ , then

- 1)  $(\bar{h})^c = \bigcup_{l=1,2,\dots,\#\bar{h}} \{(1 - \gamma_l) | p_l\}$ ;
- 2)  $\bar{h}_1 \oplus \bar{h}_2 = \bigcup_{t=1,2,\dots,\#\bar{h}_1, k=1,2,\dots,\#\bar{h}_2} \{\gamma_t + \gamma_k - \gamma_t \gamma_k | p_t p_k\}$ ;
- 3)  $\bar{h}_1 \otimes \bar{h}_2 = \bigcup_{t=1,2,\dots,\#\bar{h}_1, k=1,2,\dots,\#\bar{h}_2} \{\gamma_t \gamma_k | p_t p_k\}$ ;
- 4)  $(\bar{h})^\lambda = \bigcup_{l=1,2,\dots,\#\bar{h}} \{(\gamma_l)^\lambda | p_l\}$ ;
- 5)  $\lambda \bar{h} = \bigcup_{l=1,2,\dots,\#\bar{h}} \{1 - (1 - \gamma_l) | p_l\}$ .

**Definition 2.5**<sup>[29]</sup> Let  $\bar{h}_1(\gamma_t | p_t)$ ,  $\bar{h}_2(\gamma_k | p_k)$  be two PHFEs, then the distance between  $\bar{h}_1$  and  $\bar{h}_2$  is defined as  $d(\bar{h}_1, \bar{h}_2)$ , which satisfies the following properties:

- 1)  $0 \leq d(\bar{h}_1, \bar{h}_2) \leq 1$ ;
- 2)  $d(\bar{h}_1, \bar{h}_2) = 0$  if and only if  $\bar{h}_1 = \bar{h}_2$ ;
- 3)  $d(\bar{h}_1, \bar{h}_2) = d(\bar{h}_2, \bar{h}_1)$ .

The commonly used probability hesitant fuzzy distance measure are as follows:

- 1) The normalized Hamming distance between two PHFEs  $\bar{h}_1$  and  $\bar{h}_2$ .

$$d_H(\bar{h}_1, \bar{h}_2) = \sum_{j=1}^{\#h} | \bar{h}_1^{\sigma(j)}(\gamma_i \cdot p_i) - \bar{h}_2^{\sigma(j)}(\gamma_i \cdot p_i) |.$$

- 2) The normalized Euclidean distance between two PHFEs  $\bar{h}_1$  and  $\bar{h}_2$ .

$$d_E(\bar{h}_1, \bar{h}_2) = \sqrt{\sum_{j=1}^{\#h} | \bar{h}_1^{\sigma(j)}(\gamma_i \cdot p_i) - \bar{h}_2^{\sigma(j)}(\gamma_i \cdot p_i) |^2},$$

where  $\bar{h}_1^{\sigma(j)}$  and  $\bar{h}_2^{\sigma(j)}$  are the  $j$ th largest values in  $\bar{h}_1$  and  $\bar{h}_2$ .

**Definition 2.6** Let  $\bar{h}_1(\gamma_t | p_t)$ ,  $\bar{h}_2(\gamma_k | p_k)$  be two PHFEs, then the similarity between  $\bar{h}_1$  and  $\bar{h}_2$  is defined as  $S(\bar{h}_1, \bar{h}_2)$ , which satisfies the following properties:

- 1)  $0 \leq S(\bar{h}_1, \bar{h}_2) \leq 1$ ;
- 2)  $S(\bar{h}_1, \bar{h}_2) = 1$  if and only if  $\bar{h}_1 = \bar{h}_2$ ;
- 3)  $S(\bar{h}_1, \bar{h}_2) = S(\bar{h}_2, \bar{h}_1)$ .

By analyzing Definitions 5 and 6, it is noted that

$$S(\bar{h}_1, \bar{h}_2) = 1 - d(\bar{h}_1, \bar{h}_2).$$

Therefore, the distance measurement formula can obtain the measure of similarity in the probabilistic hesitant fuzzy sets.

**Definition 2.7**<sup>[29]</sup>  $\bar{h}_i^+ = (1 | \frac{1}{\#h}, 1 | \frac{1}{\#h}, \dots, 1 | \frac{1}{\#h})$  ( $i = 1, 2, \dots, \#h$ ) is called the probabilistic fuzzy positive ideal solution and  $\bar{h}_i^- = (0 | \frac{1}{\#h}, 0 | \frac{1}{\#h}, \dots, 0 | \frac{1}{\#h})$  ( $i = 1, 2, \dots, \#h$ ) is called the probabilistic hesitant fuzzy negative ideal solution.

Based on the aggregation operator of hesitant fuzzy sets, Xu and Zhou<sup>[28]</sup> proposed the aggregation operators of probabilistic hesitant fuzzy sets, in which the hesitant probabilistic fuzzy ordered weighted averaging (PHFOWA) operator considers both the order and the weight, and is widely used. Let  $\bar{h}_i^-$  ( $t = 1, 2, \dots, T$ ) be a collection of PHFEs,  $w = \{w_1, w_2, \dots, w_T\}$  be the

weight vector of  $\bar{h}_t$  with  $w_t \in [0, 1]$  and  $\sum_{t=1}^T w_t = 1$ ,  $p_t$  be the probability of  $\gamma_t$  in the PHFE  $\bar{h}_t$ ,  $\gamma_{\sigma(t)}$  be the  $t$ th the largest of  $\bar{h}_t$ ,  $p_{\sigma(t)}$  be the probability of  $\gamma_{\sigma(t)}$  in the PHFE  $\bar{h}_t$ , and  $w_{\sigma(t)}$  be the  $t$ th the largest of  $w$ , then

$$\begin{aligned} & \text{PHFOWA}(\bar{h}_1, \bar{h}_2, \dots, \bar{h}_T) \\ &= \oplus_{t=1}^T (w_t \bar{h}_{\sigma(t)}) = \bigcup_{\gamma_{\sigma(1)} \in \bar{h}_{\sigma(1)}, \gamma_{\sigma(2)} \in \bar{h}_{\sigma(2)}, \dots, \gamma_{\sigma(T)} \in \bar{h}_{\sigma(T)}} \left\{ 1 - \prod_{t=1}^T (1 - \gamma_{\sigma(t)})^{w_t} \mid p_{\sigma(1)} p_{\sigma(2)} \cdots p_{\sigma(T)} \right\}. \end{aligned}$$

## 2.2 Soft Sets

**Definition 2.8**<sup>[19]</sup> Let  $U$  be an initial universe set and  $E$  be a set of parameters. Let  $P(U)$  denote the power set of  $U$  and  $A \subset E$ . A pair  $(F, A)$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow P(U)$ .

**Definition 2.9**<sup>[31]</sup> For two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$ , we say that  $(F, A)$  is a soft subset of  $(G, B)$  if

- 1)  $A \subset B$ , and
- 2)  $\forall \varepsilon \in A$ ,  $F(\varepsilon)$  and  $G(\varepsilon)$  are identical approximations.

We write  $(F, A) \tilde{\subset} (G, B)$ .  $(F, A)$  is said to be a soft super set of  $(G, B)$ , if  $(G, B)$  is a soft subset of  $(F, A)$ . We denote it by  $(F, A) \tilde{\supset} (G, B)$ . Two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  are said to be soft equal if  $(F, A)$  is a soft subset of  $(G, B)$  and  $(G, B)$  is a soft subset of  $(F, A)$ .

In order to facilitate the storage of formatted data, Maji, et al.<sup>[30]</sup> proposed a tabular representation of soft sets. Clearly we can express a soft set over  $U$  as a matrix using the tabular representation. Each column of membership matrix will be represented by the vector  $\vec{F}(e_i)$ . Majumdar, et al.<sup>[31]</sup> gave the matching function based on similarity.

**Definition 2.10**<sup>[32]</sup> If  $E_1 = E_2$ , then similarity between  $(F_1, E_1)$  and  $(F_2, E_2)$  is defined as

$$S(F_1, F_2) = \frac{\sum_i \vec{F}_1(e_i) \bullet \vec{F}_2(e_i)}{\sum_i [\vec{F}_1(e_i)^2 \vee \vec{F}_2(e_i)^2]}. \quad (2)$$

where  $\vec{F}_1(e_i)$  represents the  $i$ th column vector corresponding to tabular/matrix representation of the soft set  $(F_1, E_1)$ .

Since this paper only discusses the decision-making problem under the same attribute, the case of  $E_1 \neq E_2$  is not considered.

## 2.3 Probabilistic Hesitant Fuzzy Soft Sets

Soft sets theory emphasizes the study of uncertainty and complexity from the perspective of parametrization, which can make up for the shortcomings of probabilistic hesitant fuzzy set parameterization tools and make decision-making more accurate. Therefore, this paper combines probabilistic hesitant fuzzy sets with soft sets, proposes the concept of probabilistic hesitant fuzzy soft sets (PHFSSs), and gives the following definitions.

**Definition 2.11** Let  $U$  be a universal set and  $E$  be set of parameters. Let  $\text{PHF}(U)$  denote the set of all PHFSSs defined over  $U$ . A pair  $(F, E)$  is a hesitant fuzzy soft sets if  $F(e) \in \text{PHF}(U)$  for every  $e$  in  $E$ .

**Example 1** Let  $U$  be a set of all students under consideration and  $U = \{x_1, x_2, x_3\}$ .  $E$  is a set of parameters and  $E = \{\text{virtuous, intelligent, sporty, diligent}\}$ . Then probabilistic hesitant fuzzy soft sets  $(F, A)$  defined as below gives the evaluation by a judge.

$$F(\text{virtuous}) = \{x_1 = \{0.7|0.3, 0.8|0.7\}, x_2 = \{0.5|0.3, 0.6|0.4, 0.7|0.3\}, x_3 = \{0.8|0.8, 0.9|0.2\}\},$$

$$F(\text{intelligent}) = \{x_1 = \{0.6|0.2, 0.7|0.8\}, x_2 = \{0.8|0.6, 0.9|0.4\}, x_3 = \{0.5|1\}\},$$

$$F(\text{sporty}) = \{x_1 = \{0.2|0.7, 0.3|0.3\}, x_2 = \{0.7|0.2, 0.8|0.3, 0.9|0.5\}, x_3 = \{0.5|0.6, 0.7|0.4\}\},$$

$$F(\text{diligent}) = \{x_1 = \{0.5|0.4, 0.6|0.6\}, x_2 = \{0.4|0.7, 0.6|0.3\}, x_3 = \{0.8|0.4, 0.9|0.6\}\}.$$

In order to facilitate the storage of formatted data, Maji, et al.<sup>[30]</sup> represent the soft set  $(F, A)$  as a table. Based on the tabular representation of soft sets, we give the tabular representation of probabilistic hesitant fuzzy soft sets as shown in Table 1.

**Table 1** The tabular representation of probabilistic hesitant fuzzy soft sets

$U$	$e_1$	$e_2$	$e_3$	$e_4$
$x_1$	$\{0.7 0.3, 0.8 0.7\}$	$\{0.6 0.2, 0.7 0.8\}$	$\{0.2 0.7, 0.3 0.3\}$	$\{0.5 0.4, 0.6 0.6\}$
$x_2$	$\{0.5 0.3, 0.6 0.4, 0.7 0.3\}$	$\{0.8 0.6, 0.9 0.4\}$	$\{0.7 0.2, 0.8 0.3, 0.9 0.5\}$	$\{0.4 0.7, 0.6 0.3\}$
$x_3$	$\{0.8 0.8, 0.9 0.2\}$	$\{0.5 0.1\}$	$\{0.5 0.6, 0.7 0.4\}$	$\{0.8 0.4, 0.9 0.6\}$

From the tabular representation of probability hesitant fuzzy soft set, it is found that the information of probability hesitant fuzzy soft sets can be expressed by matrix, which is more convenient for calculation. Throughout the concept of soft matrix<sup>[32]</sup>, the probability hesitant fuzzy soft matrix can be obtained, which is expressed as follows with the above example:

$$M = \begin{pmatrix} \{0.7|0.3, 0.8|0.7\} & \{0.6|0.2, 0.7|0.8\} & \{0.2|0.7, 0.3|0.3\} & \{0.5|0.4, 0.6|0.6\} \\ \{0.5|0.3, 0.6|0.4, 0.7|0.3\} & \{0.8|0.6, 0.9|0.4\} & \{0.7|0.2, 0.8|0.3, 0.9|0.5\} & \{0.4|0.7, 0.6|0.3\} \\ \{0.8|0.8, 0.9|0.2\} & \{0.5|0.1\} & \{0.5|0.6, 0.7|0.4\} & \{0.8|0.4, 0.9|0.6\} \end{pmatrix}.$$

Each probabilistic hesitant fuzzy element in the soft matrix is called probabilistic hesitant fuzzy soft element (PHFSE). Obviously, the PHFSE has the same format with PHFE in soft matrix, so some calculations related to PHFE also can apply to PHFSE.

## 2.4 TOPSIS Decision Methods on Classical Sets

Multi attribute decision-making (MCDM) refers to sorting or selecting a limited number of alternative solutions using a certain method based on existing decision information<sup>[33]</sup>. TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) is a method to evaluate the relative advantages and disadvantages of existing objects. The principle is to calculate the proximity between a limited number of evaluation objects and the idealized target with a fixed calculation rule, and then sort according to the proximity value to determine the optimal solution. At present, the TOPSIS method is constantly improved and widely used in decision-making in various fields.

In multi-attribute group decision making, we usually let  $A = \{A_1, A_2, \dots, A_m\}$  denote the set of  $m$  alternatives, where  $A_i$  denotes the  $i$ th alternative,  $i \in M$ ; let  $C = \{C_1, C_2, \dots, C_n\}$  denote the set of  $n$  attributes, where  $C_j$  denotes the  $j$ th attribute. The attribute set is divided

into two categories: Cost type and benefit type. Let  $w = (w_1, w_2, \dots, w_n)$  denote the weight vector of the attribute, where  $w_j$  is the weight of the attribute  $C_j$ , satisfying  $w_j \in [0, 1]$  and  $\sum_{j=1}^n w_j = 1$ . Let  $D = \{D_1, D_2, \dots, D_k\}$  be the set of  $k$  experts, where  $D_t$  denotes the  $t$ -th expert,  $t \in K$ . The initial decision matrix is expressed as  $X = [x_{ij}]_{m \times n}$ , where  $x_{ij}$  denotes the evaluation result of  $A_i$  for the attribute  $C_j$ , and  $x_{ij}$  is a real number.

The steps of the TOPSIS method on the traditional classical set are as follows:

- 1) Normalize the initial scoring matrix.
- 2) Construct a weight standardization matrix.
- 3) Determine the ideal solution and anti-ideal solution.
- 4) Calculate the distance measure.
- 5) Calculate the closeness of the ideal solution.

6) According to the value of the closeness degree of the ideal solution, the evaluation objectives are sorted in ascending order. The larger the closeness degree is, the better the objective is, and the maximum value is the optimal evaluation objective.

### 3 The TOPSIS Multi-Attribute Group Decision-Making Method Based on Probability Hesitant Fuzzy Soft Sets

#### 3.1 Problem Description

Suppose a probabilistic hesitant fuzzy soft set multi-attribute decision-making problem contains  $m$  alternatives and  $n$  evaluation attributes. Let  $X = \{x_1, x_2, \dots, x_m\}$  denote the alternatives,  $E = \{e_1, e_2, \dots, e_n\}$  denote the set of evaluation attributes and  $D = \{D_1, D_2, \dots, D_k\}$  denote the set of  $K$  DMs. The DMs  $D_k$  ( $k = 1, 2, \dots, K$ ) provide the evaluation values that the alternatives  $x_i$  ( $i = 1, 2, \dots, m$ ) satisfy the attributes  $e_j$  ( $j = 1, 2, \dots, n$ ) represented by HPFSEs  $\bar{h}_{ki}^j(\gamma_{ki}^j | p_{ki}^j)$ . Due to the difference in the breadth and depth of knowledge, preferences and personalities of decision makers, different weights are given to different decision makers, and denoted by  $w = \{w_1, w_2, \dots, w_k\}$  ( $k = 1, 2, \dots, K$ ).

#### 3.2 Standardization of Initial Decision Matrix of PHFSSs

In decision-making problems, the parameter set of soft set is similar to the attribute set in decision-making, and the initial full set of soft set is similar to the alternative in decision-making, so the probability hesitant fuzzy soft matrix can be directly used as the initial decision matrix.

As we all know, the calculation of distance or similarity in multi-attribute group decision-making problems generally requires the equal number of elements based on probability hesitant fuzzy elements. At present, this condition cannot be effectively met in most cases, namely the initial decision matrix often has incomplete information. In order to make any two probability hesitant fuzzy elements have the same number of elements, it is necessary to normalize them first.

Nowadays, there are two main solutions to this problem: One is to use the element with the largest or smallest membership to fill in the probability hesitant fuzzy element with fewer elements, and set its probability to 0; the second method is to delete the probability hesitant fuzzy elements with a large number of elements. Obviously, this method causes information

loss. The probability splitting algorithm proposed by Lin, et al.<sup>[34]</sup> has no effect on the score and deviation of probability hesitant fuzzy elements, and can well maintain the order relationship between the original probability hesitant fuzzy elements. Therefore, this paper uses the probability splitting algorithm to standardize the initial matrix.

### 3.3 Determination of Decision Makers' Weight Information

Based on maximizing deviation method, a maximizing score deviation (MSD) method was developed to obtain the DM's weights under the HPFE environment. The basic principle of this method is that when a decision maker and another decision maker have made a similar evaluation of all the alternatives, the decision maker cannot help us distinguish different options, so the decision maker should be assigned a smaller weight. On the contrary, the greater the evaluation gap between the schemes, the greater the weight is given.

According to the MSD method, the weight of decision maker  $D_k$  is

$$w_k = \frac{\sum_{i=1}^N \sum_{t=1}^N |s(\bar{h}_{ki}) - s(\bar{h}_{kt})|}{\sqrt{\sum_{k=1}^K (\sum_{i=1}^N \sum_{t=1}^N |s(\bar{h}_{ki}) - s(\bar{h}_{kt})|)^2}}, \quad (3)$$

where  $s(\bullet)$  is the score function of the HPFEs  $\bar{h}_{ki}$  or  $\bar{h}_{kt}$ .

Since the weight in decision-making generally satisfies normalization, the normalized weight of  $D_k$  can be obtained by substituting  $\bar{w}_k = \frac{w_k}{\sum_{k=1}^K w_k}$  into Equation (3):

$$\bar{w}_k = \frac{\sum_{i=1}^N \sum_{t=1}^N |s(\bar{h}_{ki}) - s(\bar{h}_{kt})|}{\sum_{k=1}^K \sum_{i=1}^N \sum_{t=1}^N |s(\bar{h}_{ki}) - s(\bar{h}_{kt})|}. \quad (4)$$

Thus, we can obtain the weight vector  $\bar{w} = \{\bar{w}_1, \bar{w}_2, \dots, \bar{w}_k\}$ .

### 3.4 Similarity Measure Between Probability Hesitant Fuzzy Soft Elements

Distance and similarity measure are very important in decision-making, both probability hesitant fuzzy sets and soft sets theory discussed them. The similarity calculation of soft set takes vector as the main calculation unit, which can be seen in Equation (2). The evaluation value in the soft set theory is a single value, while the evaluation value in the probability fuzzy soft sets is composed of multiple membership degrees with probability information. Therefore, this paper considers to convert each probability hesitant fuzzy element in the probability hesitant fuzzy soft matrix into a single value, and then combined with Equation (2) to calculate the similarity of the probability hesitant fuzzy soft set. Obviously, selecting the score value as a single value is simple and representative.

Based on this idea, firstly, according to Definition 2.2, the probability hesitant fuzzy soft matrix is transformed into probabilistic hesitant fuzzy soft score matrix. The probability hesitant fuzzy soft matrix in Example 1 above will be transformed into:

$$M_S = \begin{pmatrix} 0.77 & 0.68 & 0.23 & 0.56 \\ 0.6 & 0.84 & 0.83 & 0.46 \\ 0.82 & 0.5 & 0.58 & 0.86 \end{pmatrix}.$$

Secondly, the similarity  $S_S(F_1, F_2)$  between the two probabilistic hesitant fuzzy soft sets can be obtained by using the formula (2) and the two decision-maker probabilistic hesitant fuzzy



soft score matrices.

$$S_S(x_1, x_2) = \frac{\sum_{i=1}^n F_1(e_i) \cdot F_2(e_i)}{\sum_{i=1}^n (F_1(e_i)^2 \cup F_2(e_i)^2)}. \quad (5)$$

It should be noted that  $F_m(e_i)$  here represents the value of the  $m$ th row and the  $n$ th column corresponding to the probabilistic hesitant fuzzy soft score matrix. Here we illustrate this similarity calculation by an example:

**Example 2** Consider two probabilistic hesitant fuzzy soft sets  $(F, E)$  and  $(G, E)$  over  $U$ , where  $U = \{x_1, x_2\}$ ,  $E = \{e_1, e_2\}$ . Let  $A = \begin{pmatrix} \{0.9|0.5, 0.8|0.5\} & \{0.6|0.3, 0.5|0.7\} \\ \{0.6|0.7, 0.3|0.3\} & \{0.7|0.8, 0.6|0.2\} \end{pmatrix}$  and  $B = \begin{pmatrix} \{0.6|0.4, 0.4|0.6\} & \{0.9|0.2, 0.8|0.8\} \\ \{0.7|0.3, 0.3|0.7\} & \{0.4|0.9, 0.3|0.1\} \end{pmatrix}$  be their representing matrices. Firstly, we need to get probabilistic hesitant fuzzy soft score matrices  $A_S = \begin{pmatrix} 0.85 & 0.53 \\ 0.51 & 0.68 \end{pmatrix}$  and  $B_S = \begin{pmatrix} 0.48 & 0.82 \\ 0.42 & 0.39 \end{pmatrix}$ .

Then use formula (5) to calculate similarity as follows:

$$S_S(x_1^A, x_1^B) = \frac{0.85 \times 0.48 + 0.53 \times 0.82}{0.85^2 + 0.82^2} = 0.6041,$$

$$S_S(x_2^A, x_2^B) = \frac{0.51 \times 0.42 + 0.68 \times 0.39}{0.51^2 + 0.68^2} = 0.6635.$$

Previously, it was defined based on the soft set theory probability hesitant fuzzy soft set similarity. Based on the Hamming distance of the probabilistic hesitant fuzzy set, the similarity of two probabilistic hesitant fuzzy soft sets is defined below.

$$S_p(x_1, x_2) = 1 - \frac{1}{n} \left( \sum_{N=1}^n \sum_{j=1}^{\#h} \bar{h}_{MN}^1(\gamma^{\sigma(j)} \cdot p^{\sigma(j)}) - \bar{h}_{MN}^2(\gamma^{\sigma(j)} \cdot p^{\sigma(j)}) \right). \quad (6)$$

We still use the data in Example 2 to calculate the similarity  $S_p$  of the probabilistic hesitant fuzzy soft set as follows:

$$S_p(x_1^A, x_1^B) = 1 - \frac{1}{2} [|0.21 + 0.16| + |0 + 0.29|] = 1 - 0.33 = 0.67,$$

$$S_p(x_2^A, x_2^B) = 1 - \frac{1}{2} [|0.21 + 0.12| + |0.2 + 0.091|] = 1 - 0.3105 = 0.6895.$$

It can be seen from the above two results that the calculation results of the two proposed similarity methods for probabilistic hesitant fuzzy soft sets are approximately similar but different. In order to balance the results of the two methods, they are fused to obtain the average probability Hesitant fuzzy soft matrix similarity formula:

$$S_A(x_1, x_2) = \frac{1}{2} [S_S(x_1, x_2) + S_p(x_1, x_2)]. \quad (7)$$

In summary, by analyzing the similarity between soft sets and probabilistic hesitant fuzzy sets in multi-attribute group decision making, based on the idea that soft sets use vectors as similarity calculation units, this paper proposes a new probabilistic hesitant fuzzy similarity calculation formula with simpler calculation and more accurate results, and verifies the effectiveness of the proposed formula through specific examples.

### 3.5 Model Construction

Synthesize the above ideas, the multi-attribute group decision-making model based on probability hesitant fuzzy soft sets is as follows:

Step 1: Based on the evaluation value with probability information given by experts, the probability hesitant fuzzy soft initial decision matrix  $H$  is obtained.

Step 2: Use the probability splitting algorithm introduced in Subsection 3.2 to complete the data for elements with shorter lengths in the initial decision matrix, and then obtain a normalized decision matrix  $\tilde{H}$  with the same length of probabilistic hesitant fuzzy soft elements.

Step 3: Use the maximum score deviation method introduced in Subsection 3.3 to derive the weight vector  $w = \{w_1, w_2, \dots, w_k\}$  of the decision makers.

Step 4: Using the PHFOWA aggregation operator introduced in Subsection 2.1, the standard decision matrix of  $k$  decision makers is converted into a weighted decision matrix.

Step 5: Specify the standard solution according to the positive ideal solution in Definition 2.7, and calculate the similarity between each scheme and the standard solution with formulas (5)~(7).

Step 6: Comparing the similarity, the higher the similarity with the standard solution, the better the scheme. Thus, the scheme ranking is obtained.

## 4 Case Application

Excessive emissions of greenhouse gases lead to the continuous enhancement of the greenhouse effect, which has adverse effects on the global climate. Carbon dioxide is the most important part of the greenhouse gas, and reducing its emissions is regarded as the most important way to solve the climate problem. How to reduce carbon emissions has also become a global issue. In response to the challenges of global climate change, many countries and regions around the world have proposed “zero carbon” or “carbon neutrality” goals. Promoting energy transition and realizing green and low-carbon development has become a general consensus of the international community. In this context, China has proposed the dual-carbon goals of “carbon peaking” and “carbon neutrality”, and believes that there are three main bodies to achieve this goal: The state, enterprises and individuals. In order to encourage low-carbon production of enterprises, some local governments or institutions have launched activities to select “low-carbon enterprises”. In general, there are two ways to achieve the dual-carbon goal: One is to reduce carbon emissions, and the other is to increase carbon absorption. Obviously, improving the ability to reduce carbon emissions is the most direct way for enterprises. At present, coal and oil are still the main energy sources in the world, so the use of high-tech artificial carbon sequestration can reduce carbon emissions effectively, and it is also an indirect method to achieve the dual carbon goal. For enterprises, excellent management level can help enterprises optimize the production process, reduce the waste of resources and energy, so as to reduce the level of carbon emissions. Based on this, this paper selects “ability to reduce carbon emissions”, “increase carbon absorption capacity”, “form an effective low-carbon management system” as the attributes of low-carbon enterprises.

The paper assumed that after the previous rounds of selection, three enterprises are left to compete, they are  $U = \{x_1, x_2, x_3\}$ . Three experts  $D_k = \{k_1, k_2, k_3\}$  evaluate each enterprise

from three attributes  $e = \{\text{ability to reduce carbon emissions, increase carbon absorption capacity, form an effective low-carbon management system}\}$ . Next, we use the algorithm proposed above to select “low-carbon enterprises”.

Step 1: Based on the evaluation value with probability information given by three experts, the probability hesitant fuzzy soft initial decision matrix  $H_k$  is obtained as shown in Tables 2~4.

**Table 2** Decision making initial matrix  $H_1$  of  $D_1$

$U$	$e_1$	$e_2$	$e_3$
$x_1$	$\{0.6 0.8, 0.7 0.2\}$	$\{0.6 0.4, 0.8 0.6\}$	$\{0.9 1\}$
$x_2$	$\{0.5 1\}$	$\{0.7 0.7, 0.8 0.3\}$	$\{0.8 0.8, 0.9 0.2\}$
$x_3$	$\{0.7 0.4, 0.8 0.6\}$	$\{0.4 0.4, 0.6 0.6\}$	$\{0.7 0.7, 0.9 0.3\}$

**Table 3** Decision making initial matrix  $H_2$  of  $D_2$

$U$	$e_1$	$e_2$	$e_3$
$x_1$	$\{0.8 1\}$	$\{0.3 0.7, 0.7 0.3\}$	$\{0.4 0.2, 0.6 0.8\}$
$x_2$	$\{0.6 0.7, 0.7 0.3\}$	$\{0.6 0.2, 0.8 0.8\}$	$\{0.6 1\}$
$x_3$	$\{0.4 0.7, 0.5 0.3\}$	$\{0.9 1\}$	$\{0.5 0.5, 0.7 0.5\}$

**Table 4** Decision making initial matrix  $H_3$  of  $D_3$

$U$	$e_1$	$e_2$	$e_3$
$x_1$	$\{0.6 0.4, 0.7 0.6\}$	$\{0.5 0.2, 0.6 0.8\}$	$\{0.8 0.5, 0.9 0.5\}$
$x_2$	$\{0.7 0.2, 0.8 0.8\}$	$\{0.5 0.5, 0.7 0.5\}$	$\{0.6 1\}$
$x_3$	$\{0.8 1\}$	$\{0.7 0.4, 0.8 0.6\}$	$\{0.7 0.2, 0.8 0.8\}$

Step 2: Use the probability splitting algorithm introduced in Subsection 3.2 to complete the data for elements with shorter lengths in the initial decision matrix, and then obtain a normalized decision matrix  $\tilde{H}$  with the same length of probabilistic hesitant fuzzy soft elements as shown in Tables 5~7.

**Table 5** Decision making normalized matrix  $\tilde{H}_1$  of  $D_1$

$U$	$e_1$	$e_2$	$e_3$
$x_1$	$\{0.6 0.4, 0.6 0.3, 0.6 0.1, 0.7 0.2\}$	$\{0.6 0.4, 0.8 0.3, 0.8 0.1, 0.8 0.2\}$	$\{0.9 0.4, 0.9 0.3, 0.9 0.1, 0.9 0.2\}$
$x_2$	$\{0.5 0.4, 0.5 0.3, 0.5 0.1, 0.5 0.2\}$	$\{0.7 0.4, 0.7 0.3, 0.8 0.1, 0.8 0.2\}$	$\{0.8 0.4, 0.8 0.3, 0.8 0.1, 0.8 0.2\}$
$x_3$	$\{0.7 0.4, 0.8 0.3, 0.8 0.1, 0.8 0.2\}$	$\{0.4 0.4, 0.6 0.3, 0.6 0.1, 0.6 0.2\}$	$\{0.7 0.4, 0.7 0.3, 0.9 0.1, 0.9 0.2\}$

**Table 6** Decision making normalized matrix  $\tilde{H}_2$  of  $D_2$ 

$U$	$e_1$	$e_2$	$e_3$
$x_1$	$\{0.8 0.2, 0.8 0.3, 0.8 0.2, 0.8 0.3\}$	$\{0.3 0.2, 0.3 0.3, 0.3 0.2, 0.7 0.3\}$	$\{0.4 0.2, 0.6 0.3, 0.6 0.2, 0.6 0.3\}$
$x_2$	$\{0.6 0.2, 0.6 0.3, 0.6 0.2, 0.7 0.3\}$	$\{0.6 0.2, 0.8 0.3, 0.8 0.2, 0.8 0.3\}$	$\{0.6 0.2, 0.6 0.3, 0.6 0.2, 0.6 0.3\}$
$x_3$	$\{0.4 0.2, 0.4 0.3, 0.4 0.2, 0.5 0.3\}$	$\{0.9 0.2, 0.9 0.3, 0.9 0.2, 0.9 0.3\}$	$\{0.5 0.2, 0.5 0.3, 0.7 0.2, 0.7 0.3\}$

**Table 7** Decision making normalized matrix  $\tilde{H}_3$  of  $D_3$ 

$U$	$e_1$	$e_2$	$e_3$
$x_1$	$\{0.6 0.2, 0.6 0.2, 0.7 0.1, 0.7 0.5\}$	$\{0.5 0.2, 0.6 0.2, 0.6 0.1, 0.6 0.5\}$	$\{0.8 0.2, 0.8 0.2, 0.8 0.1, 0.8 0.5\}$
$x_2$	$\{0.7 0.2, 0.8 0.2, 0.8 0.1, 0.8 0.5\}$	$\{0.5 0.2, 0.5 0.2, 0.5 0.1, 0.7 0.5\}$	$\{0.6 0.2, 0.6 0.2, 0.6 0.1, 0.6 0.5\}$
$x_3$	$\{0.8 0.2, 0.8 0.2, 0.8 0.1, 0.8 0.5\}$	$\{0.7 0.2, 0.7 0.2, 0.8 0.1, 0.8 0.5\}$	$\{0.7 0.2, 0.6 0.2, 0.6 0.1, 0.6 0.5\}$

Step 3: Use the maximum score deviation method introduced in Subsection 3.3 to derive the weight vector  $w = \{w_1, w_2, \dots, w_k\}$  of the decision makers.

$$w_1 = \frac{4.08}{4.08 + 4.84 + 3.34} = 0.3328,$$

$$w_2 = \frac{4.84}{4.08 + 4.84 + 3.34} = 0.3948,$$

$$w_3 = \frac{3.34}{4.08 + 4.84 + 3.34} = 0.2724.$$

Above all, The weight vector of the three decision makers is  $w = \{0.3328, 0.3948, 0.2724\}$ .

Step 4: Using the PHFOWA aggregation operator introduced in Subsection 2.1, the standard decision matrix of  $k$  decision makers is converted into a weighted decision matrix. Since the processing of probability in PHFOWA is direct multiplication, the sum of probabilities in the aggregated probability hesitant fuzzy soft elements is not 1. At this time, the probability standardization method in Definition 2.1 can be used to convert it into probability equal to 1. Next, take three decision makers' evaluation aggregation of  $x_1$  under the  $e_1$  attribute as an example.

$$\begin{aligned} & \{1 - (1 - 0.7)^{0.3328} \times (1 - 0.8)^{0.3948} \times (1 - 0.7)^{0.2724} | 0.2 \times 0.3 \times 0.5, \\ & 1 - (1 - 0.6)^{0.3328} \times (1 - 0.8)^{0.3948} \times (1 - 0.7)^{0.2724} | 0.4 \times 0.3 \times 0.1, \\ & 1 - (1 - 0.6)^{0.3328} \times (1 - 0.8)^{0.3948} \times (1 - 0.6)^{0.2724} | 0.3 \times 0.2 \times 0.2, \\ & 1 - (1 - 0.6)^{0.3328} \times (1 - 0.8)^{0.3948} \times (1 - 0.6)^{0.2724} | 0.1 \times 0.2 \times 0.2\} \\ & = \{0.7444 | 0.03, 0.7187 | 0.012, 0.6958 | 0.012, 0.6958 | 0.004\}. \end{aligned}$$

Obviously,  $\sum_{l=1}^{\#h} p_l \neq 1$ , it needs to be standardized.

$$\tilde{p}_1 = \frac{0.03}{0.03 + 0.012 + 0.012 + 0.004} = 0.52,$$

$$\tilde{p}_2 = \tilde{p}_3 = \frac{0.012}{0.03 + 0.012 + 0.012 + 0.004} = 0.2,$$

$$\tilde{p}_4 = \frac{0.004}{0.03 + 0.012 + 0.012 + 0.004} = 0.08.$$

Thus, the three decision makers' evaluation aggregation of  $x_1$  under the  $e_1$  attribute is  $\{0.7444|0.52, 0.7187|0.2, 0.6958|0.2, 0.6958|0.08\}$ .

Similarly, it can be obtained overall decision-making matrix  $\tilde{H}$  as shown in Table 8.

**Table 8** Overall decision-making matrix  $\tilde{H}$

$U$	$e_1$	$e_2$	$e_3$
$x_1$	$\{0.7444 0.52, 0.7187 0.2, 0.6958 0.2, 0.6958 0.08\}$	$\{0.7165 0.6, 0.6039 0.16, 0.6039 0.03, 0.4698 0.21\}$	$\{0.7912 0.68, 0.7912 0.2, 0.7912 0.09, 0.7550 0.03\}$
$x_2$	$\{0.6816 0.7, 0.6433 0.2, 0.6433 0.05, 0.6016 0.05\}$	$\{0.7766 0.52, 0.7433 0.1, 0.7062 0.28, 0.6137 0.1\}$	$\{0.6824 0.68, 0.6824 0.2, 0.6824 0.09, 0.6824 0.03\}$
$x_3$	$\{0.7128 0.65, 0.6914 0.17, 0.6914 0.06, 0.6914 0.11\}$	$\{0.8084 0.63, 0.8084 0.08, 0.7860 0.06, 0.7551 0.23\}$	$\{0.8136 0.52, 0.8136 0.08, 0.6713 0.2, 0.6330 0.2\}$

Step 5: Specify the standard solution according to the positive ideal solution in Definition 2.7, and calculate the similarity between each scheme and the standard solution with formulas (5)~(7).

Since all attributes are positive, only the positive standard solution is specified as  $h^+ = \{1|0.25, 1|0.25, 1|0.25, 1|0.25\}$ . Next, taking the similarity between the probability hesitant fuzzy soft element and the positive ideal solution of  $x_1$  as an example.

$$S_1 = \frac{\frac{1}{3} \times (0.725652 + 0.643299 + 0.790114) + (1 - \frac{1}{3} \times 2.165425)}{2} = 0.4990.$$

Similarly,  $S_2 = 0.4976$ ,  $S_3 = 0.5322$ .

Step 6: Comparing the similarity, the higher the similarity with the standard solution, the better the scheme. Thus, the scheme ranking is obtained.

It can be seen from the above  $S_3 > S_1 > S_2$ , so the enterprise  $x_3$  will be awarded the title of "low carbon enterprise".

## 5 Conclusions

This article systematically investigates complex multi-attribute group decision-making problems with probabilistic hesitant fuzzy information. The specific research results are as follows:

Based on the theory of probabilistic hesitant fuzzy sets and soft sets, this paper combines probabilistic hesitant fuzzy sets with more complete evaluation information and soft sets with parameterization tools, and puts forward the concept of probabilistic hesitant fuzzy soft sets. Then, the TOPSIS multi-attribute group decision-making method based on probability hesitant fuzzy soft sets is studied, and the similarity measure of probabilistic hesitant fuzzy soft sets based on similarity measure of soft sets and distance measure of probabilistic hesitant fuzzy sets are proposed respectively. Finally, a new decision model is established by combining the

two measures, and verified by specific cases. There are two main innovations in this paper: 1) The concept of probability hesitant fuzzy soft set is proposed; 2) The probability hesitant fuzzy soft set similarity measurement is proposed and applied to the multi-attribute group decision-making problem. The proposed method provides a feasible method for multi-attribute group decision making problems.

However, there are still some problems in the research of this paper: Although there is a good connection between the two, it is currently not possible to compare them due to their different forms compared to other methods, which is one of the focus of the author's next work.

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