

## Analysis of the Impact of Inventory Shortages on the Supply Chain

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**Abstract** With the continuous development of economic globalization, especially in the face of expanding COVID-19 pandemic, the supply shortage of suppliers will directly affect the ordering strategy of enterprises, which will cause price fluctuations in the commodity market and corporate profits. We assume that the demand for the product in the market is constant, and the supply determines the product price. An EOQ model is constructed with a supply shortage under the additive supply. We find that the optimal order quantity is consistent with the classic EOQ without considering product price changes; after introducing the price function, we analyze the relationship between the product's market price and the seller's optimal profit and the supplier uncertainty. The research results show that the product's market price will increase with the increase of the out-of-stock quantity. At the same time, the uncertainty will decrease, and the seller's optimal profit will decrease as the average stock-out and uncertainty increase.

**Keywords** supply shortage; EOQ model; supply chain

### 1 Introduction

Supply uncertainty in supply chain management often occurs due to the manufacturer's internal production activities or emergencies. According to Lawrence and Shen, there are four types of supply uncertainty: Disruption, output uncertainty, capacity uncertainty, and delivery time uncertainty<sup>[1]</sup>, and the uncertainty of supply will cause difficulties for enterprises.

Take the chip shortage as an example. Since the COVID-19 outbreak, the chip shortage has plagued many industries worldwide. As the industry is highly dependent on the global supply chain, traditional vehicles require 500 to 600 chips, and the number of chips required for new energy vehicles may be as high as 2,000<sup>[2]</sup>. Affected by the shortage of chips, the production capacity of global car manufacturers has dropped significantly, and many car companies are facing the plight of reducing or even suspending production. In addition to the automotive industry, industries such as household appliances, consumer electronics, and security have also been affected by the chip shortage crisis to varying degrees. Under the superposition of multiple

factors such as insufficient production capacity, strong demand and inadequate supply chain management capabilities, the dilemma of chip shortage persists. Intel CEO Pat Gelsinger said the global chip supply shortage had battered the auto industry and other manufacturers, which will take several years to ease<sup>[3]</sup>. Confronting future with continued supply uncertainty, supply chain coordination and management has become particularly important. The chip shortage has occurred because the coronavirus pandemic has disrupted or underoperated some suppliers, resulting in a shortage of the overall supply of chips on the market. The number of chip shortages fluctuates because chip suppliers are not always outages or underpowered. In other words, the chip shortage problem is a problem of supply quantity uncertainty. As chips become scarce, market prices rise, according to supply-demand equilibrium theory. Therefore, it is very important for enterprises to formulate reasonable ordering strategies so as to reduce costs.

The classic supply chain inventory model economic order quantity (EOQ) model was proposed by Harris in 1915. Nevertheless, the necessary assumptions of the EOQ model are rarely satisfied in the real production. The supply chain often faces a variety of uncertain factors, such as perishable commodities, quantity discounts, etc. Therefore, many scholars have extended the classical EOQ model from different angles to be better applied to practical production problems. From the perspective of supply-side uncertainty, Salameh and Jaber<sup>[4]</sup> proposed an inventory model that assumes that each batch of products has quality defects with a certain probability, and the defective products can be sold at a discounted price. The result shows that the optimal order quantity increases with the average percentage of defective products. Taleizadeh, Khanbaglo and Cardenas-Barron<sup>[5]</sup> assumed an EOQ model in which defective products could be restocked for sale after being sent for inspection and repair. The paper constructed four cases and obtained the optimal solution according to the repaired product storage time. From the perspective of demand uncertainty, Maddah and Noueihed<sup>[6]</sup> assumed that the occurrence time of each unit of demand is a random variable obeying an independent and identical distribution and using the average demand to replace the deterministic demand of the original model. The optimal order quantity still satisfies the EOQ optimal order quantity formula, which shows the robustness of the input parameters of the EOQ model. Skouri<sup>[7]</sup> discussed the inventory model that allows out-of-stock and the demand rate is negatively related to the out-of-stock quantity and solved the seller's optimal ordering strategy to minimize cost. Sana<sup>[8]</sup> considered the EOQ model of sellers renting alternative warehouses under the limited capacity of their warehouses when the demand is a random variable obeying different probability distributions.

Therefore, the previous literature mainly discusses the uncertainty of the upstream and downstream of the supply chain. However, sellers not only face the supply uncertainty of the supply side but also the supply shortage in the product market will lead to price fluctuation, which will indirectly affect sellers' profits. Based on the above, this paper constructs an EOQ model with supply shortage under a fixed order cycle and introduces a supply function to solve the seller's optimal order strategy. Through model derivation, the optimal order quantity, the seller's profit and the change of the product price with the supply uncertainty are solved and analyzed. This paper enriches and expands the theoretical research of the EOQ model, and provides inspiration for inventory management decisions. What is different from previous studies is that when we consider the supply uncertainty, we only consider the shortage of

supply, not the oversupply, which leads to the complexity of our model compared with the classical supply uncertainty model. Therefore, no analytical solution can be given, only the properties of the solution can be analyzed. In addition, according to the supply and demand equilibrium theory, the supply shortage will lead to the price increase, so we consider the impact of the price increase.

The main content of this paper is as follows. In Section 2, the paper first discusses previous scholars' research on supply uncertainty and enterprises' supply chain management strategies under the supply uncertainty environment. Most of the literature mainly uses operational research optimization, mathematical modeling and game theory as the main research method. In Section 3, this paper assumes that the seller's order cycle is fixed and firstly solves the seller's optimal order quantity under the condition of a fixed price. Then, the supply function is introduced to solve the seller's optimal ordering strategy. We analyze the change in the seller's maximum profit and the price with the supply uncertainty. In Section 4, a numerical analysis is carried out on the variation of the seller's profit and product price to the average out-of-stock quantity. The conclusion is given in Section 5.

## 2 Literature Review

With the development of global economic integration, supply interruption or supply uncertainty often leads to high operating costs for industries with a high degree of integration.

Begen, Pun and Yan<sup>[9]</sup> studied the impact of supply and demand uncertainty on business costs, and their results show that when the financial loss caused by uncertainty is large, companies should prioritize reducing supply uncertainty, for example, establishing closer communication with suppliers, establish cooperative relationships with more suppliers, etc. Based on the newsvendor model, Ma, et al.<sup>[10]</sup> studied the ordering decision of loss-averse newsvendor under supply and demand uncertainty. The results show that supply uncertainty will lead decision-makers to order more products. At the same time, numerical analysis proves that supply risk has a greater negative impact on supply chain performance than demand risk. More information can help companies make more accurate decisions, reducing business risks and operating costs. However, obtaining fully accurate information is often costly and impossible in most scenarios, while misleading misinformation can even harm a company's supply chain performance. Li, Zobel and Russell<sup>[11]</sup> studied the impact of the accuracy of outage information on enterprise supply chain performance. The results show that for inaccurate outage information, the negative impact of companies with higher resilience is more serious, so for companies with high resilience, access to high-quality information is critical to effectively responding to supply disruptions.

Aiming at the risk of supply uncertainty, many scholars have studied and discussed the ordering strategy and inventory management strategy of enterprises under the situation of supply uncertainty. Among them, the replenishment strategy, as one of the classic strategies of supply chain inventory management, can relieve the pressure of insufficient inventory of enterprises when they face the risk of supply uncertainty. Yeo and Yuan<sup>[12]</sup> discussed the optimal replenishment strategy of sellers in the presence of order cancellation and supply uncertainty under the multi-period framework. The research shows that the optimal replenishment strategy

of the model has a reorder point structure. That is, there is a reorder point at which the seller chooses to order when the stock falls below this point. The emergency procurement strategy and the optimal allocation procurement strategy are widely used for managing supply disruption risks. He, Huang and Yuan<sup>[13]</sup> utilized a game theory framework to analyze the joint pricing and ordering decisions of two manufacturers in the presence of supply disruption risks. The results show that when the supplier is unreliable, the emergency procurement strategy can bring more benefits to the manufacturers than the optimal allocation procurement strategy.

Dual-source or multi-source procurement strategy can effectively help companies avoid the risk of being out of stock immediately after supply interruption due to their dependence on a single supplier, but compared with single-source procurement strategy, dual-source or multi-source procurement requires companies to have higher supply chain management and coordination capabilities and larger order requirements. Chakraborty, Chauhan and Ouhimmou<sup>[14]</sup> utilized the game theory framework to illustrate that in supply disruptions, retailers are always more willing to use alternate suppliers, even if the probability is low. The number of optimal alternate suppliers increases with the probability of outage. Assuming that there are one low-cost but unreliable supplier and the other high-cost but reliable supplier, Pal, Sana and Chaudhuri<sup>[15]</sup> constructed a multi-market multi-level supply chain model. Li and Li<sup>[16]</sup> constructed a stochastic programming model considering factors such as supply interruption, dual-source procurement and risk aversion. The results show that although unreliable suppliers have the risk of supply interruption, they are still the main suppliers of enterprises. Companies will abandon unreliable suppliers when their procurement costs are too high, or their opponents are more reliable.

Multi-source procurement is no longer limited to two suppliers with differences in supply cost and reliability. This procurement strategy incorporates the selection of suppliers into the decision-making scope and weighs the risk of supply chain disruption and enterprise costs in order to achieve optimal supply chain performance. Bagchi and Bhattacharya<sup>[17]</sup> discussed the purchasing decision problem of sellers in a multi-cycle background with uncertain demand and supply. They solved the decision variables including total order quantity, an optimal number of suppliers, and order allocation among suppliers. Meena and Sarmah<sup>[18]</sup> studied the optimal supplier quantity selection and order allocation considering the factor of supplier quantity discount under the risk of supply interruption. Compared with decentralized procurement, the results show that firms should concentrate more demand on a few suppliers to obtain higher quantity discounts and retain less risky but higher-cost suppliers as emergency alternatives. The optimal number of suppliers increases as the cost of supplier interruption risk increases.

From the perspective of supply chain coordination, supply chain contracts can effectively coordinate the upstream and downstream of the supply chain and improve the overall performance. Xue, et al.<sup>[19]</sup> studied the impact of interruption risk on corporate profits in supply chain contracts. The results show that companies are not affected by supply interruption risks under option contracts, and suppliers are solely responsible for supply risks. Under the order commitment contract, corporate profits and risks are negatively correlated. Hou, Zeng and Zhao<sup>[20]</sup> discussed how companies can implement buy-back contract with suppliers to establish long-term backup procurement partnership. Hou, Zeng and Sun<sup>[21]</sup> confirmed the importance

of backup sourcing when a major supply source disruption is inevitable. Li, Zhen, Qi and Cai<sup>[22]</sup> investigated how a core firm utilizes the option contract and the order commitment contract to share and mitigate the supply disruption risk. The results show that MS (integration of financial aid and non-delivery penalties) is the best strategy for manufacturers in most cases, but it is not a win-win strategy.

When there is supply uncertainty and supply interruption risk in the supply chain, many scholars have conducted modeling analysis from the perspectives of demand, product characteristics, supply chain structure, etc. They provide convenient ordering, inventory strategies, and inspirations for enterprises in actual production scenarios to improve supply chain performance and corporate profits.

### 3 Model

#### 3.1 EOQ Model with Supply Shortage

**Table 1** Model parameters and interpretation

Parameters	Interpretation
$Q$	number of goods ordered
$Y$	number of unfilled orders
$R$	total demand
$h$	holding cost
$K$	fixed cost
$p$	price

The economic order quantity (EOQ) model is one of the most classic inventory models in supply chain management. The model assumes that the product demand  $R$  is fixed, and minimizes the annual cost by solving the optimal order quantity  $Q$  for each order. The optimal order quantity of the EOQ model is  $Q_1^* = \sqrt{2KR/h}$ . And the optimal order quantity of EOQ model with supply uncertainty (under additive supply) is  $Q_1'^* = \sqrt{(2KR + hVAR[Y])/h + E[Y]}$ , when  $E[Y] = 0$  and  $VAR(Y) = 0$  (that is, there is no supply uncertainty), the optimal order quantity is  $Q_1'^* = Q_1^*$ <sup>[1]</sup>.

This paper considers that the seller can only place an order once within the specified order cycle when facing the uncertainty of the supplier's supply. When determining an ordering strategy, sellers need to plan for supply shortages to meet market demand for products to the greatest extent possible. Suppose that there is a supply shortage, the order quantity of the seller in each cycle is  $Q_2$ , and the supplier's supply is  $Q_2 + Y$ , where  $Y$  is a random variable distributed on the interval  $[-Q_2, 0]$ , representing the number of unfilled orders. Then the length of the seller's order cycle is  $Q_2/R$ . We assume that the seller can only release an order once in each order cycle, i.e., the seller can only place the order after  $Q_2/R$  time.

When the supply is  $Q_2 + Y$ , the seller's periodic inventory holding cost is

$$h \frac{\frac{Q_2+Y}{2}}{\frac{R}{Q_2}} = \frac{h(Q_2+Y)Q_2}{2R}. \quad (1)$$

If the market price is constant, the expected cost per period is

$$\begin{aligned}
 C &= K + \int_{-Q_2}^0 \frac{h}{2R} (Q_2 + Y) Q_2 f_Y(y) dy \\
 &= K + \frac{h}{2R} \left[ Q_2^2 \int_{-Q_2}^0 f_Y(y) dy + Q_2 \int_{-Q_2}^0 y f_Y(y) dy \right] \\
 &= K + \frac{h}{2R} (Q_2^2 + Q_2 E[Y]).
 \end{aligned} \tag{2}$$

The expected annual cost is

$$\begin{aligned}
 g(Q_2) &= \frac{C}{Q_2/R} = \frac{K + \frac{h}{2R} [Q_2^2 + Q_2 E[Y]]}{Q_2/R} = \frac{2RK + h [Q_2^2 + Q_2 E[Y]]}{2Q_2} \\
 &= \frac{RK}{Q_2} + \frac{hQ_2}{2} + \frac{hE[Y]}{2}.
 \end{aligned} \tag{3}$$

Solve the first derivative condition of  $g(Q_2)$

$$\frac{dg}{dQ_2} = -\frac{RK}{Q_2^2} + \frac{h}{2} = 0, \tag{4}$$

$$Q_2^* = \sqrt{\frac{2KR}{h}}. \tag{5}$$

When the order cycle does not vary with the supply quantity, the optimal order quantity is the same as the classical EOQ, and its size is independent of the supplier's supply uncertainty.

Now we assume that the market price of a product is determined by supply, and the supply price  $p$  is

$$p = a - bS, \tag{6}$$

where  $S$  represents the annual supply of the product, and the functional relationship shows that the larger the product supply  $S$ , the lower the market price  $p$ . In the classic EOQ model, the supply of a product equals the demand, that is,  $S = R$ . If there is a supply shortage, the annual supply of the product in the market at this time is

$$S = (Q_2 + y) \frac{R}{Q_2}. \tag{7}$$

Correspondingly, the price of the product is

$$p = a - b(Q_2 + y) \frac{R}{Q_2}. \tag{8}$$

From the above, it can be seen that the annual profit of the seller is

$$\begin{aligned}
 \pi_2 &= \int_{-Q_2}^0 \left\{ \left[ a - b(Q_2 + y) \frac{R}{Q_2} \right] (Q_2 + y) \frac{R}{Q_2} \right\} f_Y(y) dy - \int_{-Q_2}^0 \frac{h}{2} (Q_2 + y) f_Y(y) dy - \frac{R}{Q_2} K \\
 &= \frac{R}{Q_2} \int_{-Q_2}^0 \left\{ \left[ a(Q_2 + y) - b(Q_2 + y)^2 \frac{R}{Q_2} \right] \right\} f_Y(y) dy - \frac{h}{2} Q_2 - \frac{h}{2} E[Y] - \frac{R}{Q_2} K \\
 &= \frac{R}{Q_2} \left( aQ_2 + aE[Y] - bQ_2 R - 2bRE[Y] - bR \frac{E[Y^2]}{Q_2} \right) - \frac{h}{2} Q_2 - \frac{h}{2} E[Y] - \frac{R}{Q_2} K
 \end{aligned}$$

$$\begin{aligned}
&= aR + a \frac{RE[Y]}{Q_2} - bR^2 - 2b \frac{R^2 E[Y]}{Q_2} - bR^2 \frac{E[Y^2]}{Q_2^2} - \frac{h}{2} Q_2 - \frac{h}{2} E[Y] - \frac{R}{Q_2} K \\
&= aR - bR^2 + (aR - 2bR^2) \frac{E[Y]}{Q_2} - \frac{h}{2} E[Y] - bR^2 \frac{E[Y^2]}{Q_2^2} - \frac{h}{2} Q_2 - \frac{R}{Q_2} K.
\end{aligned} \tag{9}$$

If  $E[Y] = 0$  and  $\text{VAR}(Y) = 0$ ,

$$\pi_2 = aR - bR^2 - \frac{h}{2} Q_2 - \frac{R}{Q_2} K, \tag{10}$$

$$Q_2''^* = \sqrt{\frac{2KR}{h}}. \tag{11}$$

**Proposition 1** When  $E[Y] = 0$  and  $\text{VAR}(Y) = 0$ , then  $Q_2''^* = Q_1^*$ .

### 3.2 Model with Supply Curve

Then we focus on the quantity supplied when the product market is most profitable

$$\pi = pS = (a - bS)S = aS - bS^2, \tag{12}$$

$$\frac{d\pi}{dS} = a - 2bS = 0, \tag{13}$$

$$S^* = \frac{a}{2b}. \tag{14}$$

Because the optimal supply of the product is  $S^*$ , when the actual demand is greater than  $S^*$ , the seller's supply is still less than or equal to  $S^*$ , that is,  $R \leq S^*$ .

$$\pi_2 = aR - bR^2 + (aR - 2bR^2) \frac{E[Y]}{Q_2} - \frac{h}{2} E[Y] - bR^2 \frac{E[Y^2]}{Q_2^2} - \frac{h}{2} Q_2 - \frac{R}{Q_2} K, \tag{15}$$

$$\frac{d\pi_2}{dQ_2} = -\frac{h}{2} + \frac{KR}{Q_2^2} - \frac{(aR - 2bR^2)E[Y]}{Q_2^2} + \frac{2bR^2 E[Y^2]}{Q_2^3}, \tag{16}$$

$$\frac{d^2\pi_2}{dQ_2^2} = \frac{2(aR - 2bR^2)E[Y]}{Q_2^3} - \frac{2RR}{Q_2^3} - \frac{6bR^2 E[Y^2]}{Q_2^4}, \tag{17}$$

$$\lim_{Q_2 \rightarrow 0^+} \pi_2 = -\infty, \tag{18}$$

$$\lim_{Q_2 \rightarrow 0^+} \frac{d\pi_2}{dQ_2} = +\infty, \tag{19}$$

$$\lim_{Q_2 \rightarrow 0^+} \frac{d^2\pi_2}{dQ_2^2} = -\infty. \tag{20}$$

When  $R \leq S^* = \frac{a}{2b}$ ,  $a - 2bR \geq 0$ , it is obvious that

$$\frac{d^2\pi_2}{dQ_2^2} \leq 0, \quad (21)$$

$d\pi_2/dQ_2$  decreases monotonically over the feasible region.

$$\left. \frac{d\pi_2}{dQ_2} \right|_{Q_2=+\infty} = -\frac{h}{2} < 0. \quad (22)$$

**Proposition 2** *Considering the supply curve, the seller has the optimal order quantity  $Q_2^*$ .*

### 3.3 The Properties of Optimal Order Quantity, Price and Profit

Then we discuss the relationship between the optimal order quantity  $Q_2^*$  and the supplier's average number of unfilled order  $E[Y]$  and the variance  $\text{VAR}(Y)$ .

Optimal order quantity  $Q_2^*$  satisfies

$$\left. \frac{d\pi_2}{dQ_2} \right|_{Q_2=Q_2^*} = -\frac{h}{2} + \frac{KR}{Q_2^{*2}} - \frac{(aR - 2bR^2)E[Y]}{Q_2^{*2}} + \frac{2bR^2E[Y^2]}{Q_2^{*3}} = 0. \quad (23)$$

The above equation can be transformed into

$$-\frac{h}{2}Q_2^* - (aR - 2bR^2)E[Y]Q_2^* + KRQ_2^* + 2bR^2E[Y^2] = 0. \quad (24)$$

According to  $\frac{d^2\pi_2}{dQ_2^2} \leq 0$ , where  $Q_2 < Q_2^*$ , it is easy to get

$$-\frac{h}{2}(Q_2^* - 1)^3 - (aR - 2bR^2)E[Y](Q_2^* - 1) + KR(Q_2^* - 1) + 2bR^2E[Y^2] > 0, \quad (25)$$

$$\begin{aligned} & (aR - 2bR^2)E[Y](Q_2^* - 1) + \frac{h}{2}(Q_2^* - 1)^3 - KR(Q_2^* - 1) \\ & < (aR - 2bR^2)E[Y]Q_2^* + \frac{h}{2}Q_2^{*3} - KRQ_2^*, \end{aligned} \quad (26)$$

$$RK - \frac{3h}{2}Q_2^{*2} - (aR - 2bR^2)E[Y] < \frac{h}{2}(1 - 3Q_2^*). \quad (27)$$

If  $Q_2^* = 1$ ,  $-\frac{h}{2}Q_2^{*3} - (aR - 2bR^2)E[Y]Q_2^* + KRQ_2^* + 2bR^2E[Y^2]$  can be transformed into

$$-\frac{h}{2} - (aR - 2bR^2)E[Y] + KR + 2bR^2E[Y^2], \quad (28)$$

$$R \leq a/2b \implies aR - 2bR^2 > 0, \quad (29)$$

$$E[Y] < 0 \implies (aR - 2bR^2)E[Y] < 0, \quad (30)$$

$$KR - \frac{h}{2} = \frac{h}{2} \left( \frac{2KR}{h} - 1 \right) = \frac{h}{2}(Q_2^{*2} - 1) = 0. \quad (31)$$



So, we have

$$-\frac{h}{2} - (aR - 2bR^2)E[Y] + KR + 2bR^2E[Y^2] > 0, \quad (32)$$

$$Q_2^* > 1. \quad (33)$$

Therefor

$$RK - \frac{3h}{2}Q_2^{*2} - (aR - 2bR^2)E[Y] < 0. \quad (34)$$

Taking the derivation of  $\text{VAR}(Y)$  and  $E[Y]$  on both sides of the optimal order quantity equation

$$\frac{dQ_2^*}{d\text{VAR}(Y)} = \frac{-2bR^2}{RK - \frac{3}{2}hQ_2^{*3} - (aR - 2bR^2)E[Y]} > 0, \quad (35)$$

$$\frac{dQ_2^*}{dE[Y]} = \frac{(aR - 2bR^2)Q_2^* - 4bR^2E[Y]}{RK - \frac{3}{2}hQ_2^{*3} - (aR - 2bR^2)E[Y]} < 0. \quad (36)$$

It can be concluded that the seller's optimal order quantity  $Q_2^*$  increases with the increase of the uncertainty of the supplier's supply shortage ( $\text{VAR}(Y)$ ), and decreases with the decrease of the supplier's average shortage ( $-E[Y]$ ).

**Proposition 3** *The seller's optimal order quantity  $Q_2^*$  increases with the increase of the uncertainty of the supplier's supply shortage ( $\text{VAR}(Y)$ ), and decreases with the decrease of the supplier's average shortage ( $-E[Y]$ ).*

$$\frac{dE[p_2^*]}{d\text{VAR}(Y)} = -bRE[Y] \frac{-1}{Q_2^*} \frac{dQ_2^*}{d\text{VAR}(Y)} = \frac{bRE[Y]}{Q_2^*} \frac{dQ_2^*}{d\text{VAR}(Y)} < 0. \quad (37)$$

The product's market price decreases as the uncertainty of the supplier's out-of-stock quantity increases, because as the uncertainty of the supplier's out-of-stock quantity increases, the seller will increase the order quantity of the product per cycle  $Q_2^*$ , thereby reducing the price of the product in the market.

**Proposition 4** *The market price of the product decreases with the increase of the uncertainty of the supplier's shortage quantity.*

$$E[p_2^*] = a - b(Q_2^* + E[Y]) \frac{R}{Q_2^*} = a - bR - \frac{bRE[Y]}{Q_2^*}, \quad (38)$$

$$\frac{dE[p_2^*]}{dE[Y]} = -bR \left( \frac{1}{Q_2^*} - \frac{E[Y]}{Q_2^{*2}} \frac{dQ_2^*}{dE[Y]} \right) = -\frac{bR}{Q_2^*} \left( 1 - E[Y] \frac{dQ_2^*}{dE[Y]} \right). \quad (39)$$

If  $1 - E[Y]dQ_2^*/dE[Y] > 0$ , then  $dE[p_2^*]/dE[Y] < 0$ .

**Proposition 5** *The average price of a product decreases as the number of suppliers out of stock ( $-E[Y]$ ) decreases.*

In the end, we consider the variation of the seller's maximum profit with the uncertainty of the supplier's out-of-stock quantity and the average out-of-stock quantity when the seller's optimal order quantity is  $Q_2^*$ .

$$\pi_2^* = aR - bR^2 + (aR - 2bR^2) \frac{E[Y]}{Q_2^*} - \frac{h}{2} E[Y] - bR^2 \frac{E[Y^2]}{Q_2^{*2}} - \frac{h}{2} Q_2^* - \frac{R}{Q_2^*} K, \quad (40)$$

$$\begin{aligned} \frac{d\pi_2^*}{d\text{VAR}(Y)} &= - \frac{(aR - 2bR^2) E[Y]}{Q_2^{*2}} \frac{dQ_2^*}{d\text{VAR}(Y)} - bR^2 \left( \frac{1}{Q_2^{*2}} - \frac{2E[Y^2]}{Q_2^{*3}} \frac{dQ_2^*}{d\text{VAR}(Y)} \right) \\ &\quad - \frac{h}{2} \frac{dQ_2^*}{d\text{VAR}(Y)} + \frac{RK}{Q_2^{*2}} \frac{dQ_2^*}{d\text{VAR}(Y)} \\ &= \left( - \frac{(aR - 2bR^2) E[Y]}{Q_2^{*2}} + \frac{2bR^2 E[Y^2]}{Q_2^{*3}} - \frac{h}{2} + \frac{RK}{Q_2^{*2}} \right) \frac{dQ_2^*}{d\text{VAR}(Y)} - \frac{bR^2}{Q_2^{*2}}, \end{aligned} \quad (41)$$

$$\begin{aligned} \frac{d\pi_2^*}{dE[Y]} &= (aR - 2bR^2) \left( \frac{1}{Q_2^*} - \frac{E[Y]}{Q_2^{*2}} \frac{dQ_2^*}{dE[Y]} \right) - \frac{h}{2} - bR^2 \left( \frac{2E[Y]}{Q_2^{*2}} - \frac{2E[Y^2]}{Q_2^{*3}} \frac{dQ_2^*}{dE[Y]} \right) \\ &\quad - \frac{h}{2} \frac{dQ_2^*}{dE[Y]} + \frac{RK}{Q_2^{*2}} \frac{dQ_2^*}{dE[Y]} \\ &= \left( - \frac{(aR - 2bR^2) E[Y]}{Q_2^{*2}} + \frac{2bR^2 E[Y^2]}{Q_2^{*3}} - \frac{h}{2} + \frac{RK}{Q_2^{*2}} \right) \frac{dQ_2^*}{dE[Y]} \\ &\quad + \frac{aR - 2bR^2}{Q_2^*} - \frac{h}{2} - \frac{2bR^2 E[Y]}{Q_2^{*2}}. \end{aligned} \quad (42)$$

Since the optimal order quantity  $Q_2^*$  satisfies

$$\left. \frac{d\pi_2}{dQ_2} \right|_{Q_2=Q_2^*} = -\frac{h}{2} + \frac{KR}{Q_2^{*2}} - \frac{(aR - 2bR^2) E[Y]}{Q_2^{*2}} + \frac{2bR^2 E[Y^2]}{Q_2^{*3}} = 0, \quad (43)$$

$$\frac{d\pi_2^*}{d\text{VAR}(Y)} = -\frac{bR^2}{Q_2^{*2}} < 0, \quad (44)$$

$$\frac{d\pi_2^*}{dE[Y]} = \frac{aR - 2bR^2}{Q_2^*} - \frac{h}{2} - \frac{2bR^2 E[Y]}{Q_2^{*2}}. \quad (45)$$

It can be concluded that the seller's maximum profit decreases as the uncertainty of the supplier's out-of-stock quantity increases. If  $(aR - 2bR^2)/Q_2^* - 2bR^2 E[Y]/Q_2^{*2} > 0$ , the seller's maximum profit increases as the supplier's average out-of-stocks  $(-E[Y])$  decreases.

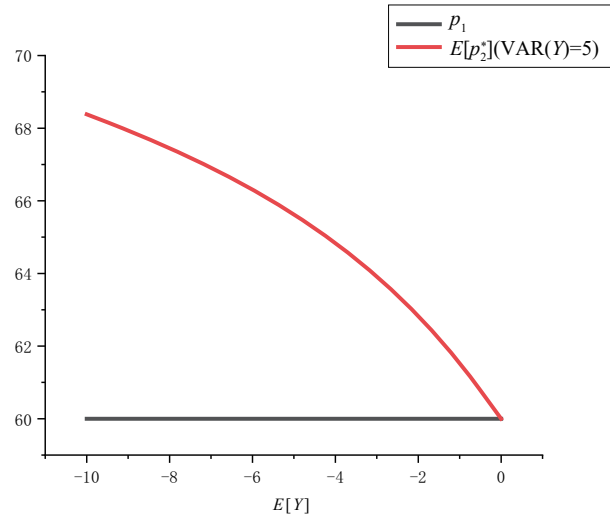
**Proposition 6** *The seller's maximum profit decreases as the uncertainty of the supplier's out-of-stock quantity increases. If  $(aR - 2bR^2)/Q_2^* - 2bR^2 E[Y]/Q_2^{*2} > 0$ , the seller's maximum profit increases as the supplier's average out-of-stocks  $(-E[Y])$  decreases.*

Our model considers the seller's optimal ordering strategy when the order cycle is fixed and the product price fluctuates with the supply. Through model derivation, it can be concluded that the seller's optimal order quantity  $Q_2^*$  increases with the uncertainty of the supplier's supply shortage ( $\text{VAR}(Y)$ ) and decreases as the supplier's average out-of-stocks  $(-E[Y])$  decreases. Further analysis shows that the product market price increases with the uncertainty of the shortage, and the seller's maximum profit increases and decreases with the uncertainty.

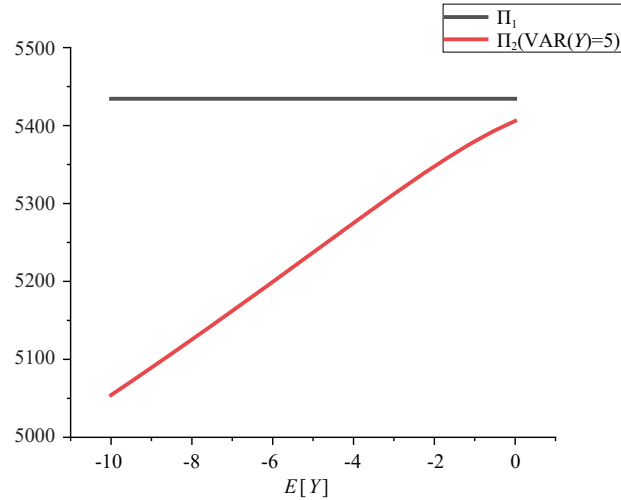
#### 4 Data and Case Analysis

In this part, we compare the influence of the supplier's average out-of-stock quantity ( $-E[Y]$ ) on the market average price ( $E[p_2^*]$ ) of the product and the seller's maximum profit ( $\pi_2^*$ ) under different parameter settings, where  $\pi_1$  and  $p_1$  denote the price of the product and the seller's profit without shortage.

Assuming that an existing car manufacturer  $M$  needs chips for producing 100 cars, the number of chips required for each car is normalized to 1 unit, so the total chip demand is  $R = 100$  units, and its supplier  $S$  has supply uncertainty, Uncertainty variance  $\text{VAR}(Y) = 5$ . The purchase cost of  $M$  is  $K = 80$ , ignoring the production process, the product price has a linear relationship with the market supply  $p = 110 - 0.5Q$ , and the changes of  $E[p_2^*]$  and  $\pi_2^*$  are shown in the figure below.



**Figure 1**  $K = 80, a = 110, b = 0.5, R = 100, h = 20, E[p_2^*]$  changes with  $E[Y]$



**Figure 2**  $K = 80, a = 110, b = 0.5, R = 100, h = 20, \pi_2^*$  changes with  $E[Y]$

The results show that when the parameters are close to reality, the product's market price will increase with the increase of the supplier's average out-of-stock quantity ( $-E[Y]$ ), while the seller's maximum profit will decrease accordingly.

With the deepening of the uncertainty of the supply shortage, the seller will adopt an ordering strategy of appropriately increasing the order quantity, thereby reducing the product's market price. In contrast, the increase in the average shortage of the supplier will lead to product shortages. The market supply falls, and the price rises. At the same time, we can observe that the annual profit of the manufacturer decreases monotonically as the uncertainty of the supplier's supply increases. Among them, the seller's profit increases with the decrease of the supplier's average out-of-stock quantity and decreases with the increase of the variance of the supplier's out-of-stock quantity.

In the data analysis section, we also consider the case that the supply curve is  $p = a - bQ^v$ , ( $v \geq 1$ ). According to the calculation formula of the seller's expected profit, the seller's expected profit at this time can be obtained as

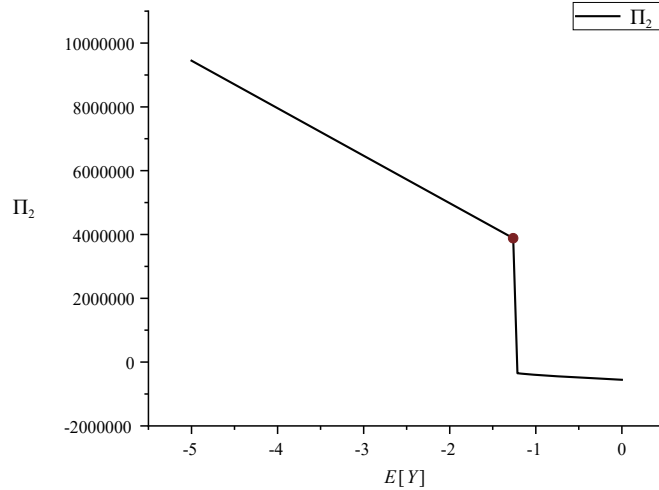
$$\begin{aligned}\pi'_2 &= \int_{-Q_2}^0 \left\{ \left[ a - b \left[ (Q_2 + y) \frac{R}{Q_2} \right]^v \right] (Q_2 + y) \frac{R}{Q_2} \right\} f_Y(y) dy - \int_{-Q_2}^0 \frac{h}{2} (Q_2 + y) f_Y(y) dy - \frac{R}{Q_2} K \\ &= \frac{aR}{Q_2} (Q_2 + E[Y]) - b \frac{R^{v+1}}{Q_2^{v+1}} E[(Q_2 + Y)^{v+1}] - \frac{hE[Y]}{2} - \frac{h}{2} Q_2 - \frac{RK}{Q_2}.\end{aligned}\quad (46)$$

We set  $v = 2$ , and  $p = a - bQ^2$ , then we have

$$\pi'_2 = \frac{aR}{Q_2} (Q_2 + E[Y]) - \frac{bR^3}{Q_2^3} (Q_2^3 + 3Q_2^2 E[Y] + 3Q_2 E[Y^2] + E[Y^3]) - \frac{hE[Y]}{2} - \frac{h}{2} Q_2 - \frac{RK}{Q_2}. \quad (47)$$

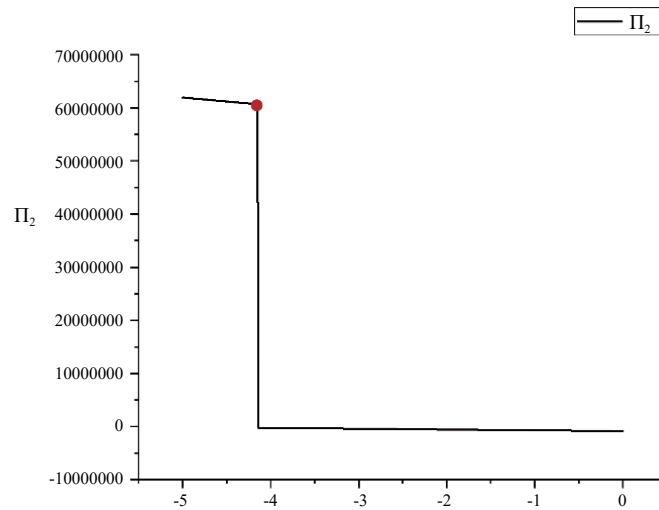
Assuming  $R = 100$ ,  $K = 80$ ,  $h = 20$ ,  $a = 110$ ,  $b = 0.5$ , and in addition, we discuss how the seller's optimal profit  $\pi_2^*$  varies with the average out-of-stock quantity ( $-E[Y]$ ) in two cases:

① the second and third origin moments of the random variable  $Y$  are  $E[Y^2] = 5$  and  $E[Y^3] = -20$ , respectively. The variation of  $\pi_2^*$  with the average backorder ( $-E[Y]$ ) is shown in Figure 3.



**Figure 3**  $E[Y^2] = 5$ ,  $E[Y^3] = -20$ ,  $\pi_2^*$  changes with  $E[Y]$

② The second and third central moments of the random variable  $Y$  are  $\text{VAR}(Y) = 5$ ,  $E[(Y - E[Y])^3] = 0$ , respectively. The variation of  $\pi_2^*$  with the average backorder ( $-E[Y]$ ) is shown in Figure 4.



**Figure 4**  $E[Y^2] = 5$ ,  $E[Y^3] = 0$ ,  $\pi_2^*$  changes with  $E[Y]$

As can be seen from the figure, as the average out-of-stock quantity decreases, the supplier's optimal profit decreases. This is due to the fact that when the market price of the product decreases exponentially with the increase of supply, sellers will instead tend to keep products out of stock, so that the price of the product remains in a high position, thereby obtaining a higher expected return. As can be seen from the red marks in Figures 3 and 4, the optimal profit of the seller  $\pi_2^*$  will decline rapidly after reaching a certain inflection point, and the profit will decrease to negative.

Take the automotive chip market as an example. Due to the pandemic and the tight supply and shortage of chips, the prices in the chip field also fluctuate one after another, with the overall increase ranging from 10% to 40%<sup>[23]</sup>, which can indirectly prove Proposition 5: The average price of a product decreases as the number of suppliers out of stock ( $-E[Y]$ ) decreases.

At the same time, due to the lack of chips, the global auto industry is generally under pressure to stop production and reduce production. The lack of cores directly leads to some car companies having to reduce or delay the delivery date of new cars. This part indirectly confirms Proposition 3: the seller's optimal order quantity  $Q_2^*$  increases with the increase of the uncertainty of the supplier's supply shortage ( $\text{VAR}(Y)$ ), and decreases with the decrease of the supplier's average shortage ( $-E[Y]$ ). Due to the prolonged delivery cycle of new cars and the tight market, the domestic second-hand car market has become hot. According to the latest data released by the China Automobile Dealers Association, in September, the scale of second-hand car transactions in China was around 1.6 million. The annual second-hand car transaction volume will exceed 16 million<sup>[24]</sup>. The hot second-hand car market shows that the demand does not decrease with the shortage of chips, so the supply is less than the demand, which conforms with the hypothesis of supply shortage in this paper.

It can be seen from the chip shortage dilemma of the global automobile market that under the impact of the epidemic, the shortage of the global chip market has caused various car companies to be unable to complete annual production targets to varying degrees. At the same time, with the rising prices of chips and products in the automotive market, the negative impact of supply uncertainty has been further expanded. At a time when the shortage of chip supply still exists and will continue, companies need to optimize their procurement and inventory strategies further to maximize supply chain performance.

## 5 Conclusion

This paper studies the optimal ordering strategy, product-market price and profit of sellers in the EOQ model with a supply shortage. Suppose that the order cycle does not change with the supply quantity. In that case, the optimal order quantity is the same as the classic EOQ, which has nothing to do with the supplier's out-of-stock quantity and supply uncertainty. Considering the supply function, the seller's optimal order quantity increases with the uncertainty of supply shortage, so the product price decreases. Moreover, the seller's maximum profit decreases with uncertainty. This paper analyzes the relationship between product market price, seller's maximum profit and supplier's average out-of-stock quantity through numerical simulation. The simulation results show that the price of the product will decrease with the decrease of the out-of-stock quantity, and the profit of the supplier will increase with the decrease of the out-of-stock quantity. Our work extends the EOQ model and provides advice on how companies can navigate uncertain supply.

This paper considers sellers' best order strategy under additive supply shortage. As for the future research direction, we believe that this topic can be extended from the perspective of multiplicative supply shortage, which the arrival quantity of the seller is  $YQ$ , where  $0 \leq Y \leq 1$ , to study the influence of supply shortage on the optimal ordering decision of the seller. In addition, this paper only considers the two-stage supply chain of one seller and one retailer, and does not consider whether there are alternative products in the market. Therefore, the future research direction can also be analyzed from the perspective of competition among multiple sellers.

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